Ken Intriligator, PITP lecture 1 homework, July 2006

Recall,

$$W_{dyn} = \begin{cases} (N_c - N_f) \left(\frac{\Lambda^{3N_c - N_f}}{\det M}\right)^{1/(N_c - N_f)} & N_f < N_c \\ 0 \quad (\text{on } \mathcal{M}_{cl}) & N_f \geq N_c. \end{cases} \tag{1}$$

And for $N_f = N_c + 1$

$$W_{dyn} = -\frac{1}{\Lambda^{2N_c - 1}}(M_{f\tilde{g}} B^f \tilde{B}^{\tilde{g}} - \det M), \tag{2}$$

which is irrelevant in the IR.

1. Consider $SU(N_c)$ SQCD with $N_f < N_c$, and give all of the flavors a mass, $W_{tree} = m^{f\tilde{g}} M_{f\tilde{g}}$. The full exact superpotential is $W_{exact} = W_{dyn} + W_{tree}$, with W_{dyn} given in (1). Verify that there are N_c supersymmetric vacua, which are particular values of $\langle M \rangle$ which solve $\frac{\partial W_{exact}}{\partial M_{f\tilde{g}}} = 0$. Verify that these $\langle M \rangle$ can be written as $\langle M_{f\tilde{g}} \rangle = \frac{\partial W(m)}{\partial m^{f\tilde{g}}}$, with $W(m) = N_c \left(\det m \Lambda^{3N_c - N_f}\right)^{1/N_c} e^{2\pi i k/N_c}$, with $k = 1 \ldots N_c$. These expressions for $\langle M \rangle$ in the N_c susy vacua are applicable also for $N_f \geq N_c$ massive flavors.

2. Consider $SU(N_c)$ with $N_f = N_c + 1$ massless flavors. The dimension of the moduli space of vacua is $\dim_C(\mathcal{M}) = 2N_f N_c - (N_c^2 - 1) = N_f^2$. As described in point 14, there are $N_f^2 + 2N_f$ massless fields at the origin (all the mesons and baryons). Verify that the superpotential (2) has susy vacua which satisfy the constraints found in the previous exercise. This shows that the correct $\mathcal{M}_{cl}$ is reproduced away from the origin.

3. 't Hooft anomalies are computed from the anomaly triangle diagrams, but with global currents at each vertex. They represent obstructions to gauging global symmetries. 't Hooft argued that they must be constant along RG flows. In particular, they must match for the UV and IR fermion spectrum. Verify 't Hooft anomaly matching is satisfied, with the same 't Hooft anomalies for $SU(N_c)$ with $N_f = N_c + 1$ flavors in the UV, and the composite fields $M_{f\tilde{g}}$ and B^f and $\tilde{B}^{\tilde{f}}$ in the IR. The global symmetries (ABJ anomaly free) are $SU(N_f) \times SU(N_f) \times U(1)_B \times U(1)_R$, and every cubic combinations of them gives a 't Hooft anomaly which must match. The charges of the IR fields are computed using $M = Q\tilde{Q}$, $B = Q^{N_c}$, and $\tilde{B} = \tilde{Q}^{N_c}$.

4. Start from $N_f = N_c + 1$ and consider giving a large mass m to one of the flavors. At low energy, we can decouple the massive flavor and recover the theory with $N_f = N_c$ flavors. The dynamical scales of the two theories are related by $\Lambda_{low}^{2N_c} = m\Lambda^{2N_c - 1}$

(as found by matching the running coupling at scale m). In the effective theory (2), we add $W_{tree} = mM_{N_f N_f}$ to account for the mass m for the N_f-th flavor. Solve the F-term equations for M_{i,N_f}, $M_{N_f,i}$, B^{N_f}, $\tilde{B}^{N_f}$. Show that all these fields are massive (their expectation value is fixed). Show that the remaining fields have $W_{low} = (W_{dyn} + mM_{N_f N_f})|_{\langle M_{iN_f}\rangle,\ldots} = 0$, but that the remaining massless fields are constrained by $\det M - B\tilde{B} = \Lambda_{low}^{2N_c}$; this is Seiberg's quantum deformed moduli space constraint for $N_f = N_c$.

5. Start from (2) and add $W_{tree} = \mathrm{Tr} mM$, where m is a mass matrix for all N_f flavors. Verify that there are N_c supersymmetric vacua, given by $\langle B^f \rangle = \langle \tilde{B}_f \rangle = 0$, and $\langle M_{f\tilde{g}} \rangle$ given by the same expression found in exercise 1, simply extrapolated to $N_f = N_c + 1$.