Dynamical SUSY Breaking, Lecture 1

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1. Introduction

The topic of these lectures is dynamical supersymmetry breaking. A theory that dynamically breaks supersymmetry has

1. No explicit SUSY breaking, $\mathcal{L} = \mathcal{L}_{susy}$.
2. The vacuum spontaneously break susy.
3. The susy breaking is related to some dynamical scale

$$\Lambda = M_{cutoff} e^{-c/g(M_{cutoff})^2} \ll M_{cutoff}.$$

As pointed out long ago by Witten, this can naturally lead to hierarchies. E.g. the weak scale m_W can be dynamically generated, by dimensional transmutation, explaining why $m_W/m_{pl} \sim 10^{-17}$. Spontaneous SUSY breaking retains the SUSY cancellation of quadratic divergences to the Higgs mass. Models of dynamical SUSY breaking (DSB) can be useful for model building, e.g. coupled to the MSSM. Here we will not discuss model building, but will discuss simple examples and general aspects of DSB.

A question of interest for these lectures is *"How generic is dynamical supersymmetry breaking in the landscape of all possible susy gauge theories, and in the landscape of string vacua.* As will be reviewed, theories with no susy vacua seem to be very non-generic. However, theories with meta-stable DSB vacua could be much more common.

2. Low energy effective field theory

Supersymmetry breaking is an issue about the IR dynamics of a theory. Asymptotically free gauge theories are simple, free theories in the UV, but become strongly coupled in the IR; so it is generally a tough problem to determine their IR dynamics. Asymptotic freedom says that the microscopic fields interact weakly for energies $E \gg \Lambda$, where Λ is the dynamically generated scale. The running gauge coupling gets strong for energies $E \sim \Lambda$. Below that scale, the strong gauge coupling often actually simplifies the problem, by binding together the strongly interacting microscopic quarks and gluons into more manageable macrscopic degrees of freedom. Determining the dynamics at $E \sim \Lambda$ is tough, but things can simplify in the extreme IR, $E \ll \Lambda$.

The IR dynamics is described by a low-energy effective field theory. The problem is to determine the degrees of freedom of this effective theory, and their interactions. Sometimes symmetries help. E.g. for non-supersymmetric $\mathcal{N} = 0$ $SU(N_c)$ QCD, with N_f massless quark flavors, $q_f \in N_c$ and $\tilde{q}_f \in N_c$, the $SU(N_f)_L \times SU(N_f)_R \to SU(N_f)_D$ SSB of $\langle q_f \tilde{q}^f \rangle \sim \Lambda^3$ implies that there are massless goldstone boson pions in $SU(N_f)_L \times SU(N_f)_R / SU(N_f)_D$, interacting via the chiral lagrangian, which is IR free.

For supersymmetric gauge theories with a classical moduli space (for $W_{tree} = 0$), the classical moduli (i.e. the gauge invariant monomials of chiral superfields) are (at least some of) the light fields of the low-energy effective field theory. Their interactions are governed by a supersymmetric effective lagrangian, e.g. an effective Kahler potential and superpotential. The superpotential is tightly constrained by the symmetries (the Kahler potential is not).

3. Our main example: $\mathcal{N} = 1$ supersymmetric SQCD

As discussed in the tutorial, the gauge group is $SU(N_c)$, with matter fields $Q_f \in \mathbf{N_c}$ and $\tilde{Q}_f \in \overline{\mathbf{N_c}}$. There are equal numbers of fundamentals to satisfy the condition of no gauge anomalies $\mathrm{Tr} T^3 = 0$, where the trace is over all matter fields.

Aside: this matter content satisfies the no gauge anomaly condition by being "vector-like," meaning that all matter can be given mass terms, here via $W_{tree} = m^{f\tilde{g}} Q_f \tilde{Q}_{\tilde{g}}$. The fields Q and $\tilde{Q}$ have opposite sign $SU(N_c)$ generators, so $\mathrm{Tr} T^3 = 0$. A "chiral" theory satisfies the constraint more non-trivially, e.g. $SU(5)$ with matter $A \in \mathbf{10}$ and $\tilde{Q} \in \overline{\mathbf{5}}$ also has total gauge anomaly $\sim \mathrm{Tr} T^3 = 0$.

3.1. The symmetries

The gauge and [global] symmetries are

	$SU(N_c)$	$[SU(N_f)_L$	$SU(N_f)_R$	$U(1)_B$	$U(1)_R$	$U(1)_A]$
Q	N_c	N_f	$\cdot$	1	$1 - \frac{N_c}{N_f}$	1
$\tilde{Q}$	$\overline{N}_c$	$\cdot$	N_f	-1	$1 - \frac{N_c}{N_f}$	1
M	$\cdot$	N_f	N_f	0	$2(1 - \frac{N_c}{N_f})$	2
$\Lambda^{3N_c - N_f}$	$\cdot$	$\cdot$	$\cdot$	0	0	$2N_f$
W_{eff}	$\cdot$	$\cdot$	$\cdot$	0	2	0

(3.1)

3.2. Anomalies, instanton zero modes and charges

ABJ anomaly of global current: $\partial_\mu J^\mu = (\#)\mathrm{Tr}F\tilde{F}/32\pi^2$. Can compute # from the triangle diagram, with the global current J^μ at one vertex and the gauge fields at the other two. Can also compute # from mathematics: index of Dirac operator = number of fermion zero modes in instanton background. Each $SU(N_c)$ fundamental or anti-fundamental has $\# = 1$ zero mode. Each $SU(N_c)$ adjoint, i.e. the gauginos, has $\# = 2C_2(G) = 2N_c$ zero modes. (E.g. $SU(N_c)$ SYM: the classical $U(1)_R$ is explicitly broken, by instantons, to Z_{2N_c} (which is then spontaneously broken to Z_2 by $\langle S\rangle = \Lambda^3 e^{2\pi ik/N_c}$, $k = 1\ldots N_c$).) Each $SU(N_c)$ fundamental, e.g. each flavor of Q and $\tilde{Q}$ has 1 zero mode.

The instanton amplitude goes like $e^{-S_{inst}} = e^{-8\pi^2/g^2+i\theta}$ and the 1-loop running of the (holomorphic) gauge coupling is

$$e^{-8\pi^2/g^2(\mu)+i\theta} = \left(\frac{\Lambda}{\mu}\right)^{b_1} = \left(\frac{\Lambda}{\mu}\right)^{3N_c-N_f},$$

where b_1 is the coefficient of the 1-loop beta-function.

The $U(1)_R$ charge assignment in (3.1) is chosen to be anomaly free, which is equivalent to the fact that the instanton 't Hooft vertex, which is the vertex with the $2N_c$ gaugino zero modes λ and $2N_f$ quark zero modes, ψ_Q and $\psi_{\tilde{Q}}$ has net $U(1)_R$ charge zero (don't forget that $R(\psi_Q) = R(Q) - 1$, and the entry in the table (3.1) is $R(Q)$).

The $U(1)_A$ symmetry in (3.1) is anomalous, as the $2N_f$ quark zero modes in the instanton background has net $U(1)_A$ charge $2N_f$. Rather than thinking of $U(1)_A$ as explicitly broken by instantons, we can think of it as being spontaneously broken by assigning the instanton charge to the instanton amplitude, i.e. to $\Lambda^{3N_c-N_f}$, as in the table (3.1), and thinking of Λ as the expectation value of a background chiral superfield. Now the effective superpotential must respect $U(1)_A$ too – this is the notion of selection rules, as in the Stark effect. The effective superpotential must also be holomorphic in $\Lambda^{3N_c-N_f}$, since the dynamics doesn't know that it's not a background chiral superfield. This observation is due to Seiberg.

3.3. The exact dynamical superpotential for SQCD

Using the symmetries (3.1), we find

$$W_{dyn} \propto \left(\frac{\Lambda^{3N_c-N_f}}{\det M}\right)^{1/(N_c-N_f)} \tag{3.2}$$

For $N_f < N_c$, this expression makes a lot of sense. Recall that the gauge group is Higgsed to $SU(N_c - N_f)$. For $N_f = N_c - 1$, the gauge group is completely Higgsed, and then there are finite action (constrained) instantons, and indeed precisely in this case (3.2) is proportional to the 1-instanton amplitude. For $N_f < N_c - 1$, (3.2) is instead associated with gaugino condensation in the unbroken $SU(N_c - N_f)$ – that is the reason for the fractional power in (3.2).

For $N_f < N_c$, the expression (3.2) moreover satisfies the boundary condition that we know from asymptotic freedom, that $W_{dyn} \to 0$ for $M/\Lambda^2 \to \infty$. However, for $N_f > N_c$ (3.2) seemingly does not satisfy this asymptotic freedom boundary condition.

For $N_f < N_c$, the classical moduli space is lifted non-perturbatively, for $N_f \geq N_c$ it is not:

$$W_{dyn} = \begin{cases} (N_c - N_f)\left(\frac{\Lambda^{3N_c - N_f}}{\det M}\right)^{1/(N_c - N_f)} & N_f < N_c \\ 0 \quad (\text{on } \mathcal{M}_{cl}) & N_f \geq N_c. \end{cases} \tag{3.3}$$

As you'll see in the exercise, there is a meaning to the analog of (3.2) for $N_f = N_c + 1$ (and higher) if it is properly interpreted. In any case, as you'll also see in the exercise the statement in (3.3) is still strictly correct.

What happens to the singularity of $\mathcal{M}_{cl}$ for $N_f \geq N_c$? Answer given by Nati. Seiberg. For $N_f = N_c$, the symmetries allow the space $\mathcal{M}_{cl}$ to be smoothed, $\mathcal{M}_{quantum} \neq \mathcal{M}_{cl}$ (by instantons) in the quantum theory. For $N_f > N_c$, the symmetries do not allow any smoothing: $\mathcal{M}_{quantum} = \mathcal{M}_{classical}$ = singular. The singularity corresponds to new massless fields there.

For $N_f = N_c + 1$, the quantum theory at the origin is given by the following *low energy effective field theory*. There are IR free fields $M_{f\tilde{g}}$, B^f, and $\tilde{N}^{\tilde{g}}$, "mesons and baryons," with *no constraints imposed*. The Kahler potential is smooth (and approximately canonical) for these fields. Evidence for this is the non-trivial 't Hooft anomaly matchings satisfied by these fields. They interact via the superpotential

$$W_{dyn} = -\frac{1}{\Lambda^{2N_c - 1}}(M_{f\tilde{g}} B^f \tilde{B}^{\tilde{g}} - \det M), \tag{3.4}$$

which is irrelevant in the IR.