Integrability in AdS/CFT

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Abstract

The AdS/CFT duality claims the equivalence of $\mathcal{N} = 4$ extended supersymmetric gauge theory and IIB string theory on the $AdS_5 \times S^5$ background. Unfortunately, quantitative tests of this conjecture are usually prevented by its strong/weak nature. One way this problem might eventually be overcome is to make use of the apparent *integrability* of both models in the 't Hooft limit. Here, the central object of investigation is the *spectrum*. The term 'spectrum' refers to the anomalous *scaling dimensions* of local operators in the conformal gauge theory or to the *energies* of string vibration modes in string theory. Powerful tools for the investigation of the spectrum, related to *integrable spin chains*, have been discovered in the recent few years.

This series of four lectures shall give an introduction into the subject of integrability in AdS/CFT and present the main methods and techniques which are in use to study it. The focus will be on the Bethe ansatz technique for solving integrable spin chains which arise in gauge theory. We shall discuss two of its forms, their derivation and applications. Of particular interest will be the thermodynamic limit where contact to classical strings can be made. Finally, methods to study the classical spectrum of integrable string sigma models are discussed and compared to gauge theory.

Review Papers

There exist a fair amount of review papers of the subject of integrability in AdS/CFT and related topics. These contain further references to the technical literature. Let me summarise their main content here:

[1] An first (and up-to-date) introduction to spinning strings. Covers explicit solutions, the Bethe ansatz and its thermodynamic limit.

[2] A review of spinning strings solutions.

[3] A talk mostly on the spectrum and spectral curves.

[4] A classic: Introduction to the algebraic Bethe ansatz for SU(2) spin chains.

[5] A dictionary on one-loop gauge theory with an introduction to the algebraic Bethe ansatz. Some higher-loop corrections.

[6] A talk on the Bethe ansatz for $\mathcal{N} = 4$ SYM at one and higher loops.

[7] On effective field theory (coherent states) methods for integrable spin chains.

[8] On integrability in gauge theories with non-maximal supersymmetry and QCD.

It might be useful to have printed copies of a few of these reviews.

Outline

This is the (tentative) outline of the lectures:

Lecture 1: Overview & Introduction

- spectrum of AdS/CFT
- How (not) to compute string energies?
- How (not) to compute anomalous dimensions?
- dilatation generator
- planar limit and spin chains
- spin chains
- spectral curves for classical strings
- Bethe equations for perturbative gauge theory
- thermodynamic limit
- some elementary papers: [9, 2, 1]

Lecture 2: Solving Classical Sigma Models (String Theory)

- the worldsheet
- Lax connection, monodromy
- analytic structure, poles, branch points
- moduli, string modes
- some elementary papers: [10, 3, 11]

Lectures 3 & 4: Solving Integrable Spin Chains (Gauge Theory)

- coordinate space Bethe ansatz
- scattering phase/matrix
- factorized scattering, Yang-Baxter equation
- conventional Bethe equations
- R-matrix formalism
- algebraic Bethe ansatz
- transfer matrix, Baxter equation
- thermodynamic limit
- integral equations, spectral curve
- quantum consistency
- comparison to strings
- some elementary papers: [4, 5, 6, 12, 1]

References

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[12] M. Staudacher, *"The factorized S-matrix of CFT/AdS"*, JHEP 0505, 054 (2005), hep-th/0412188.