Hadron collider kinematics

Express momenta in terms of

transverse mom. $\quad p_T = \sqrt{p_x^2 + p_y^2}$

rapidity $\quad y = \frac{1}{2} \ln \frac{E+p_z}{E-p_z}$

azimuthal angle $\quad \varphi$

$$p^\mu = (E_T \cosh y, p_T \cos\varphi, p_T \sin\varphi, E_T \sinh y)$$

$$p^2 = m^2 \iff E_T^2 = p_T^2 + m^2$$

p obtained by boost with rapidity y from momentum $\perp$ to beam

$$p^\mu = \Lambda^\mu{}_\nu \tilde{p}^\nu$$ with

$$\tilde{p}^\mu = (E_T, p_T \cos\varphi, p_T \sin\varphi, 0)$$ and

$$\Lambda = \Lambda(y) = \begin{pmatrix} \cosh y & 0 & 0 & \sinh y \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \sinh y & 0 & 0 & \cosh y \end{pmatrix}$$

$$= \exp\left(y \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \right)$$

Rapidities are additive for boosts along the z-axis

$$\Lambda(y')\,\Lambda(y) = \Lambda(y'+y)$$

$$\Rightarrow \Lambda^{\mu}{}_{\nu}(y')\, p^{\nu} = \left(E_T \cosh(y'+y),\ \vec{P}_T,\ E_T \sinh(y'+y)\right)$$

- Massless particles

$$p^{\mu} = E\left(1,\ \sin\theta\cos\varphi,\ \sin\theta\sin\varphi,\ \cos\theta\right)$$

$$\Rightarrow y = \frac{1}{2}\ln\left(\frac{1+\cos\theta}{1-\cos\theta}\right) \equiv \eta \quad \text{pseudorapidity}$$

- c.m. momentum

$$p^{\mu} = E_b\left(x_1+x_2,\ 0,\ 0,\ x_1-x_2\right)$$

$$\Rightarrow y_{c.m.} = \frac{1}{2}\ln\frac{x_1}{x_2}$$

For a particle with

rapidity y^* in c.m. frame
rapidity y in lab frame

$$\Rightarrow y = y^* + y_{c.m.}$$

Added benefit: Use of (pseudo)rapidity flattens t- or u-channel poles in differential cross sections

$$t = -\frac{s}{2}(1-\cos\theta^*)$$

$$u = -\frac{s}{2}(1+\cos\theta^*)$$

$$\frac{d\sigma}{d\cos\theta^*} \sim \frac{1}{ut} \sim \frac{1}{\sin^2\theta^*} = \frac{d\eta}{d\cos\theta^*}$$

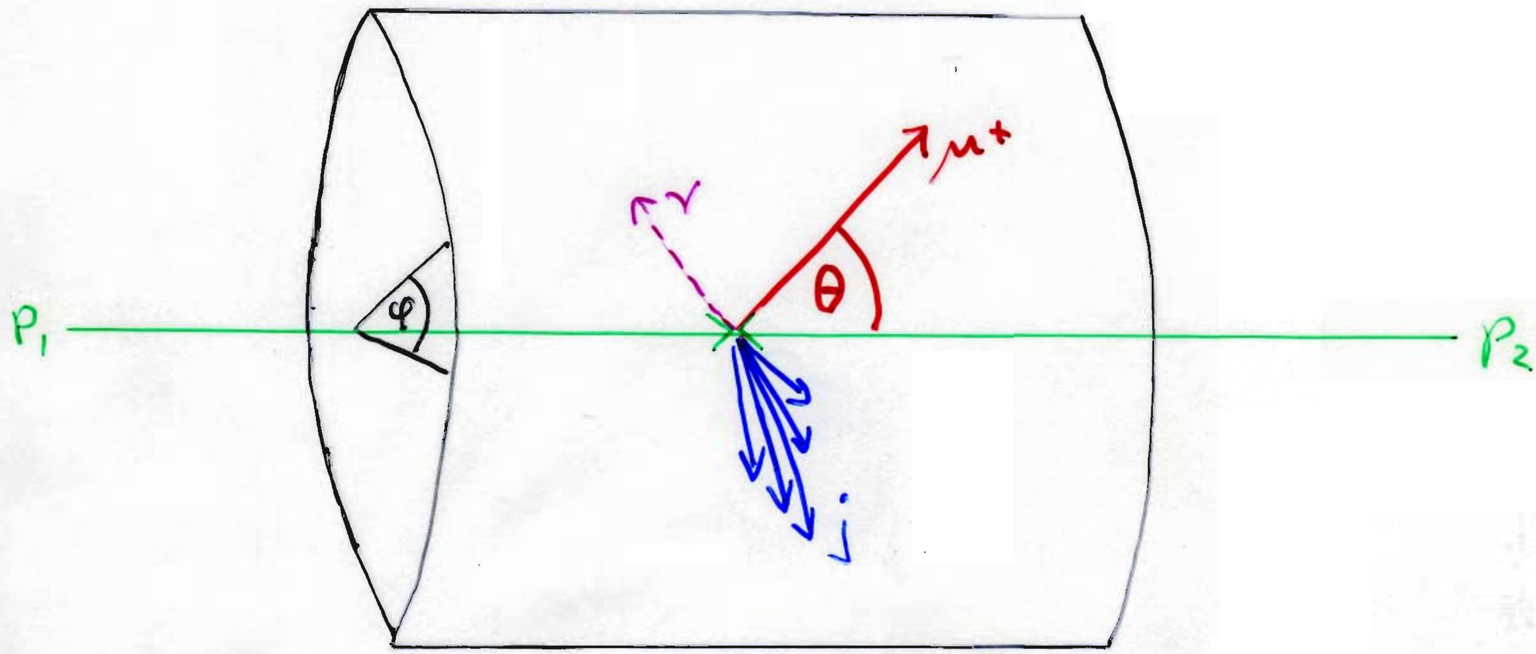
$$\Rightarrow \frac{d\sigma}{d\eta^*} \sim \text{constant}$$

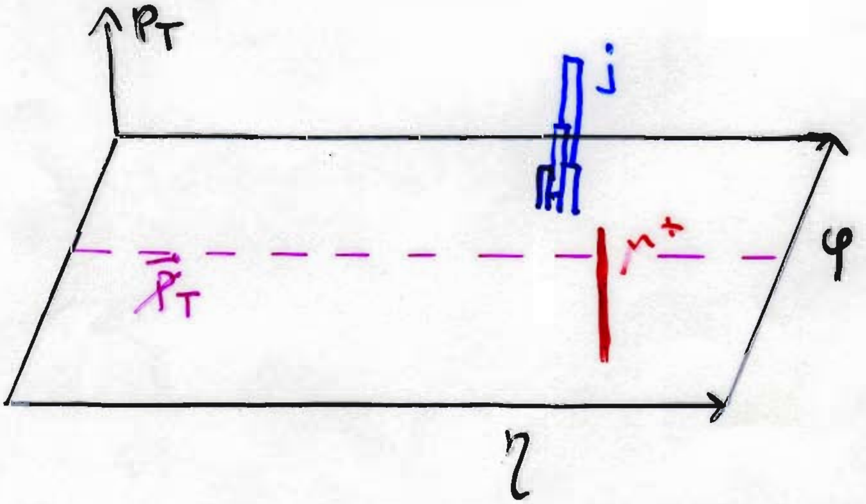
Phase space measure

$$d\phi_n(p_a+p_b \to p_1+\dots+p_n) = (2\pi)^4\,\delta^4(p_a+p_b-\sum_i p_i) * \prod_{i=1}^{n} \frac{d^3\vec{p}_i}{(2\pi)^3\,2E_i}$$

$$\frac{d^3\vec{p}}{E} = p_T\,dp_T\,dy\,d\varphi = \frac{1}{2}\,dp_T^2\,dy\,d\varphi$$

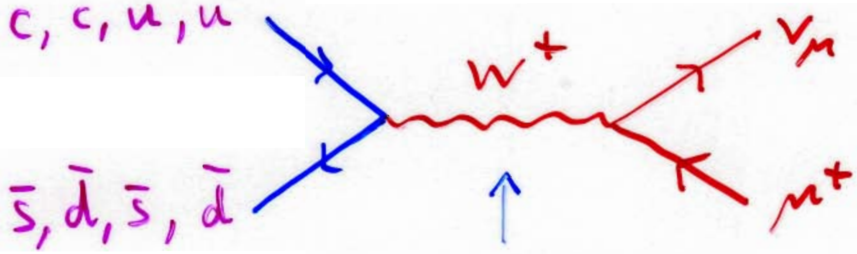
Legoplot variables : η, φ, p_T





Example: $pp \to W^+X \to \mu^+\nu_\mu X$

subprocesses: $q\bar{q}' \to W^+ \to \mu^+\nu_\mu$



$$W: \quad \frac{-ig^{\mu\nu}}{\hat{s} - m_W^2 + i m_W \Gamma_W}$$ Breit-Wigner

Kinematics:

$$\left.\begin{aligned} q &= x_1 \frac{\sqrt{s}}{2}(1, 0, 0, 1) \\ \bar{q}' &= x_2 \frac{\sqrt{s}}{2}(1, 0, 0, -1) \end{aligned}\right\} \text{zero } p_T$$

transverse momentum balance: $p_{T\mu^+} = p_{T\nu}$

$$\mu = p_T(\cosh\eta, \cos\varphi, \sin\varphi, \sinh\eta)$$

$$\nu_\mu = p_T(\cosh\eta', -\cos\varphi, -\sin\varphi, \sinh\eta')$$

same transverse mom. in c.m. frame

$$\mu^* = \frac{m_W}{2}(1, \sin\theta^*\cos\varphi, \sin\theta^*\sin\varphi, \cos\theta^*)$$

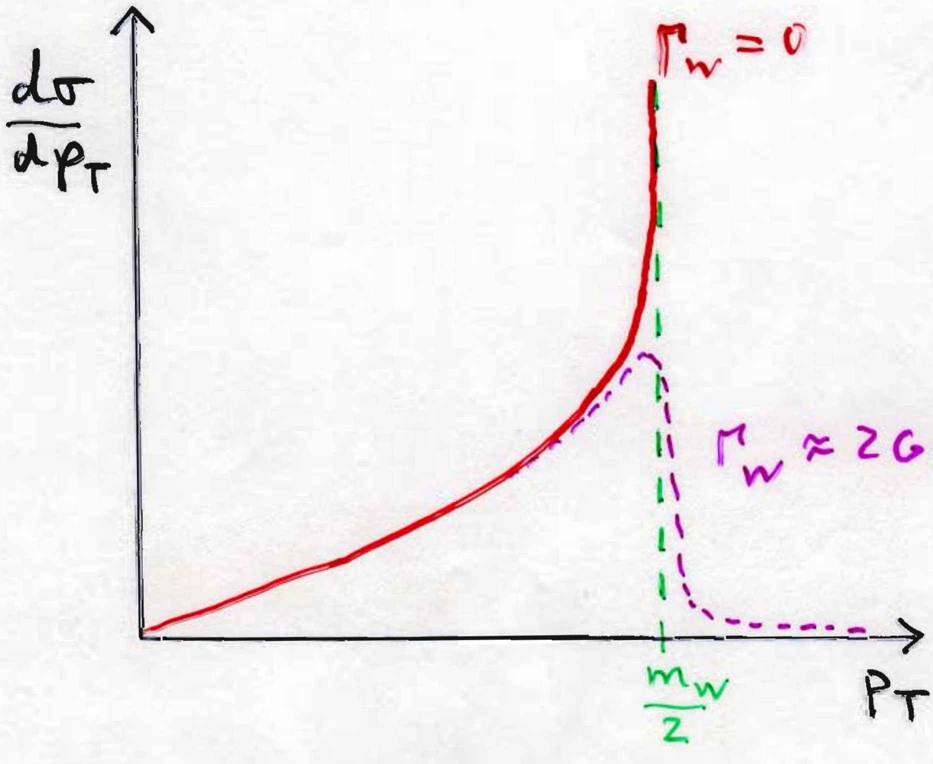
$$\Rightarrow \quad p_T = \frac{m_W}{2} \sin\theta^*$$

$$\cos\theta^* = \sqrt{1-\sin^2\theta^*} = \sqrt{1 - \frac{4p_T^2}{m_W^2}}$$

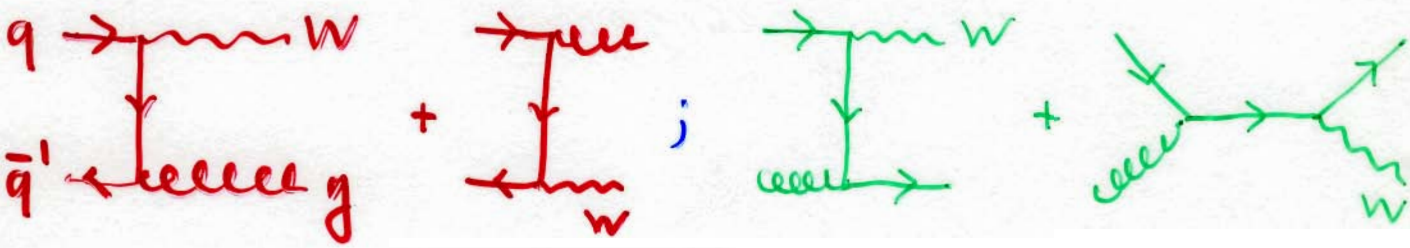
differential cross section

$$\frac{d\hat{\sigma}}{dp_T^2} = \frac{d\hat{\sigma}}{d\cos\theta^*}\left|\frac{d\cos\theta^*}{dp_T^2}\right| = \frac{d\hat{\sigma}}{d\cos\theta^*}\,\frac{2/m_W}{\sqrt{m_W^2 - 4p_T^2}}$$

Jacobian factor $\longrightarrow$ Jacobian peak at $p_T = m_W/2$



Beyond finite width effect, gluon emission (higher order processes) leads to $p_{TW} > 0$



Resummation of multiple soft gluon emission $\rightarrow$ Sudakov suppression

$\Rightarrow$ $p_{TW} = 0$ never happens

compare: $p_T(Z)$ @ Tevatron

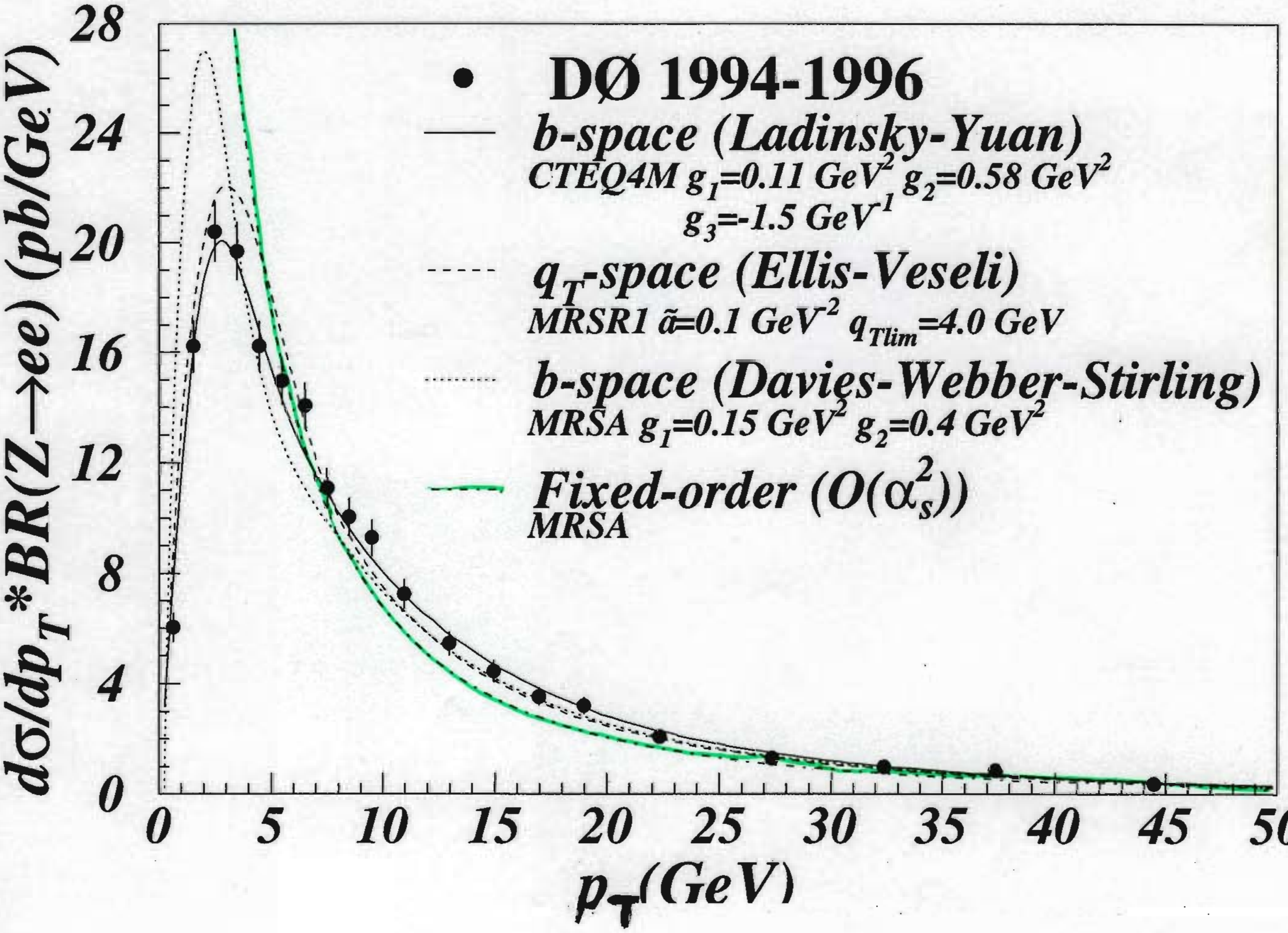
Choose observables which minimize effect of soft gluon radiation!

At least: infrared safe observables

Use transverse mass for $W \to \ell\nu$

$$m_T = \sqrt{(p_{Te} + p_{T\nu})^2 - (\vec{p}_{Te} + \vec{p}_{T\nu})^2}$$

Z transverse momentum distribution at the Teratron



Jacobian peak gets smeared due to

- finite W/Z width
- detector resolution
- parton emission

DØ, PRL 98; p_T(hadrons) < 15 GeV; ⇒ $m_W = 80.44 \pm 0.12$

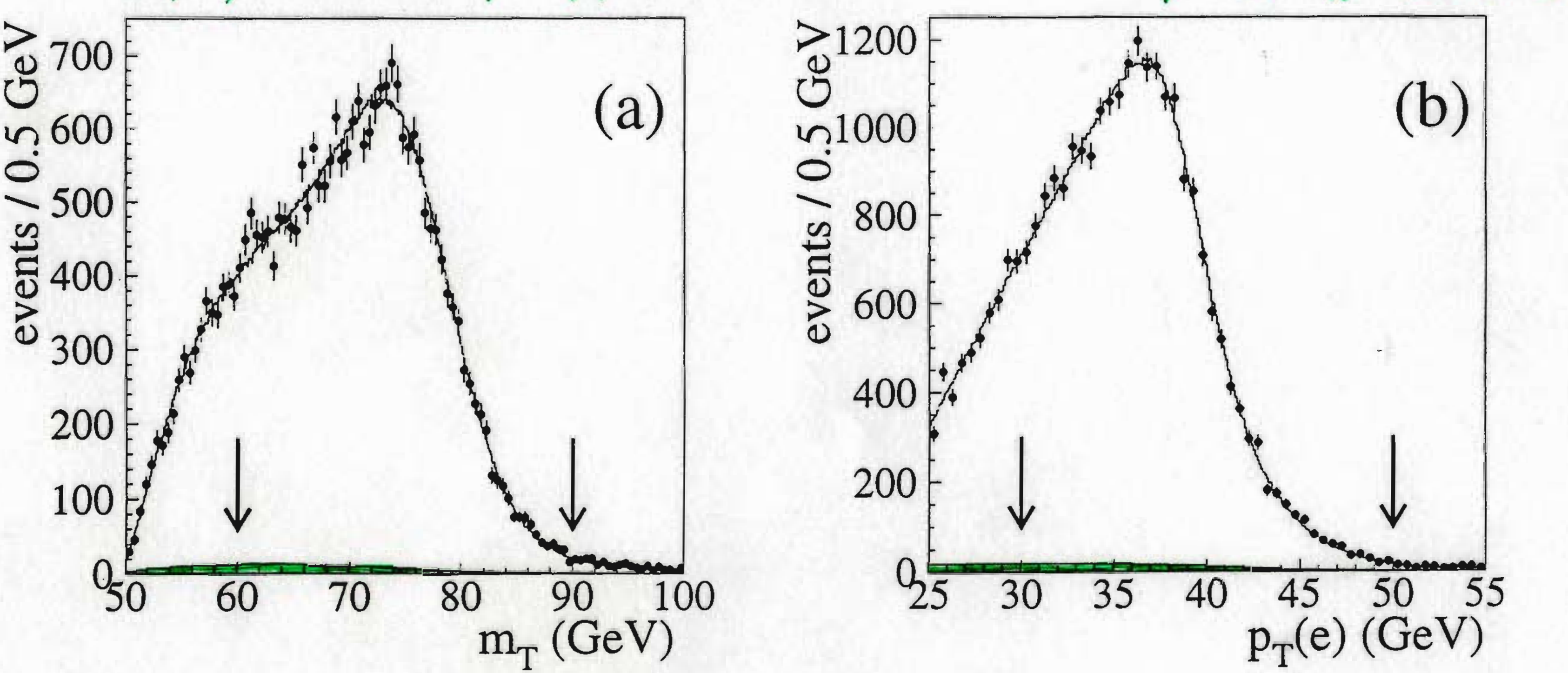


FIG. 3. Spectra of (a) m_T and (b) $p_T(e)$ from the data (•), the fit (—), and the backgrounds (shaded). The arrows indicate the fit windows.

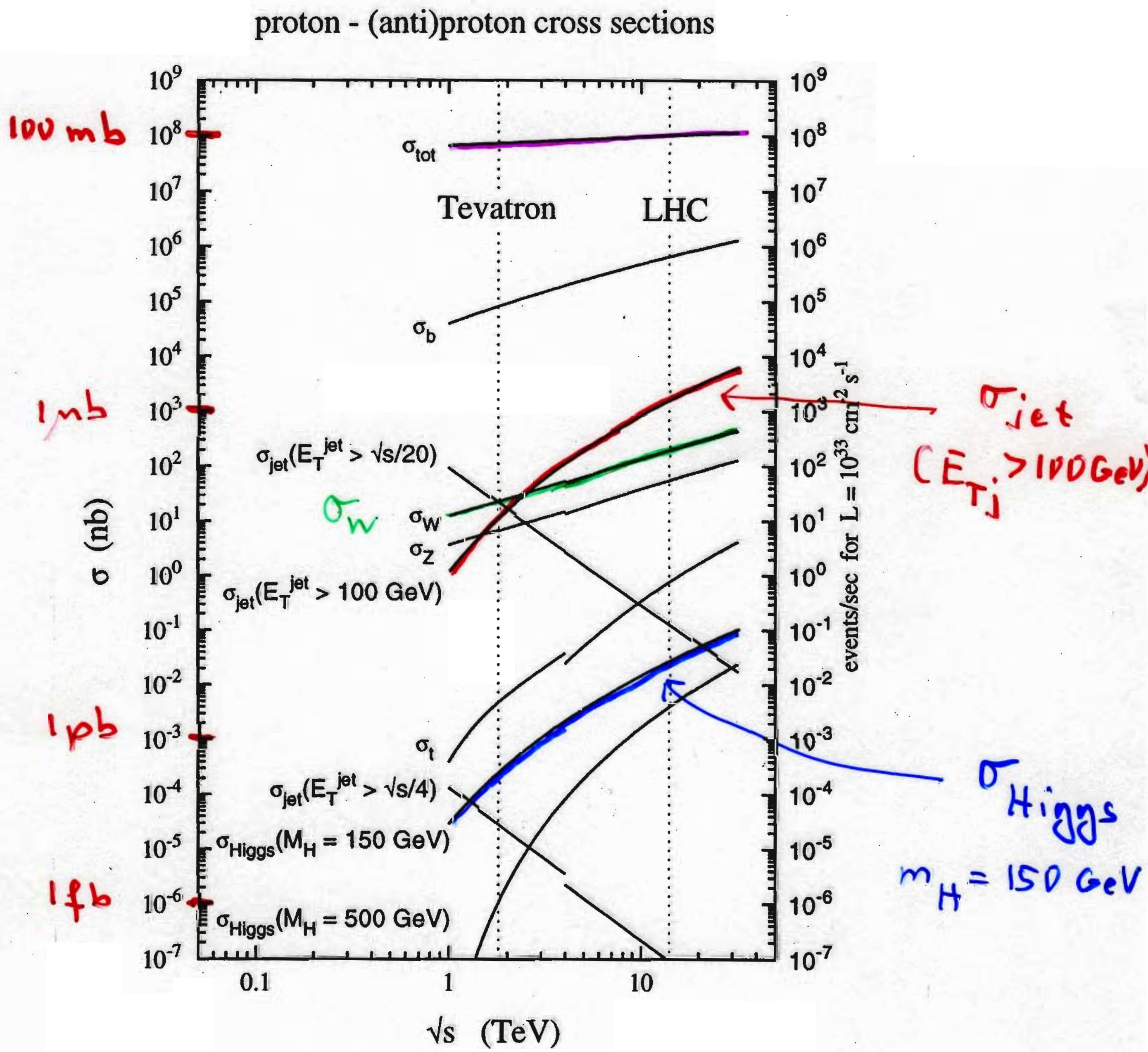
Jets at hadron colliders

cross sections

jet algorithms

- cone
- k_T

Typical hadron collider cross sections

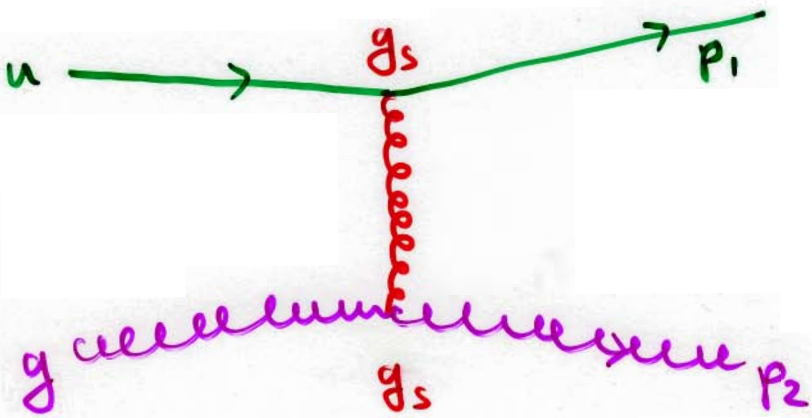


$\Rightarrow$ study jets

- as background
- as tool

Partons and jets

Rutherford scattering of quarks & gluons



$\hat{s} = (p_1 + p_2)^2$

$\vec{p}_1 + \vec{p}_2 \parallel$ z-axis

$p_T = E_1 \sin\theta_1$
$\quad = E_2 \sin\theta_2$

High energy : $p_T = 50 \dots 500$ GeV at Tevatron

$$\frac{g_s^2}{4\pi} = \alpha_s(p_T) \approx 0.1$$

perturbation theory valid

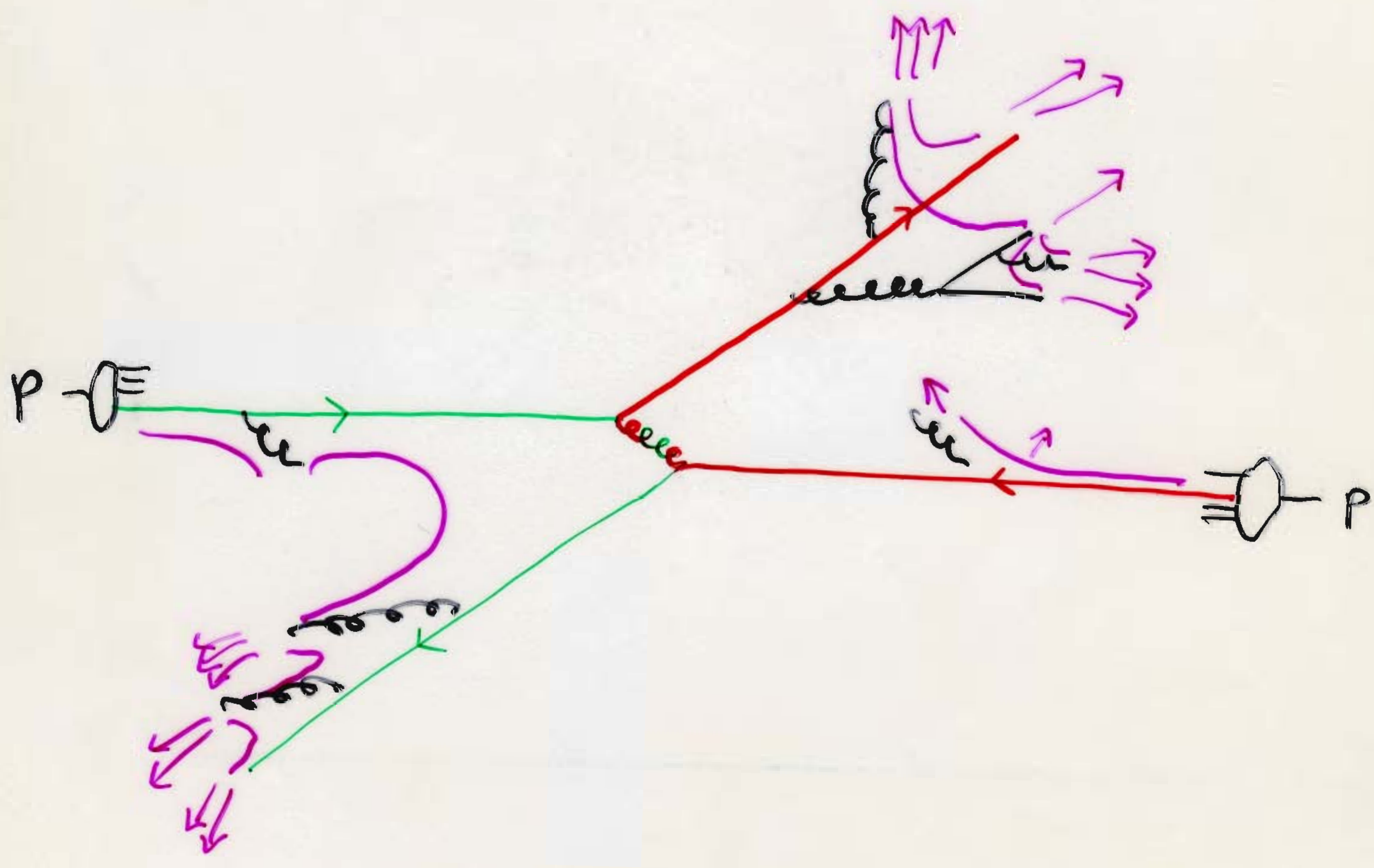
asymptotic freedom :

scattered partons $\approx$ free particles

Hard scattering process : perturbative QCD

parton shower } PYTHIA, Herwig,

hadronization } Isajet



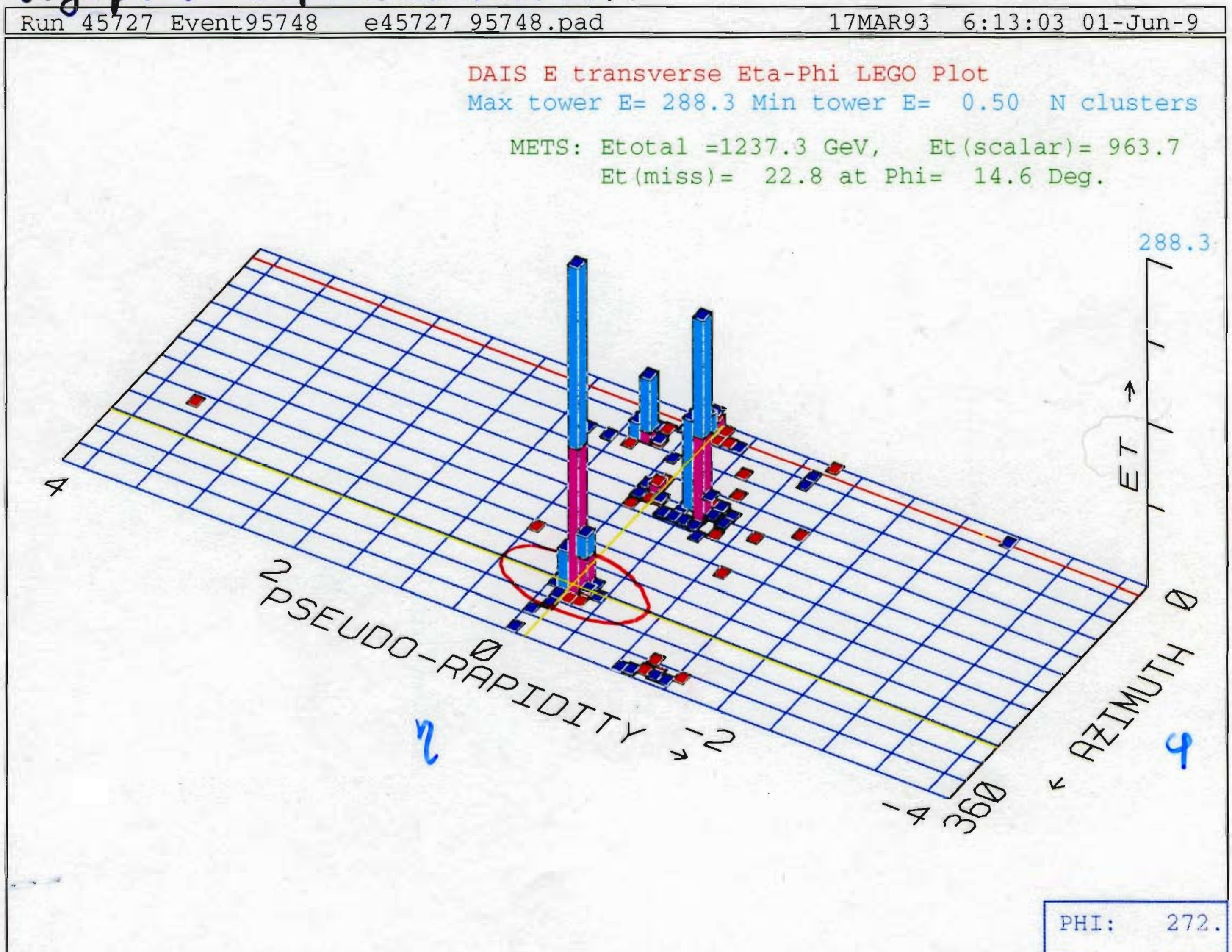
Tevatron run I:

Event with highest visible energy in CDF (1993)

$$E_{vis} = 1237\ \text{GeV}$$

$$E_T = 964\ \text{GeV}$$

legoplot representation



pseudorapidity: $\eta = \frac{1}{2} \ln \frac{1+\cos\theta}{1-\cos\theta}$

$$E_T(\text{jet}) = \sum_{R<0.7} E \sin\theta$$

At <u>tree level</u> we can identify

parton $\equiv$ jet

parton direction = jet direction (η, φ)

parton p_T = jet E_T

number of jets with small E_T explodes

$\Leftrightarrow$ infrared catastrophe for gluon emission, collinear divergences

$\Rightarrow$ require

$E_T > E_{Tj}^{min}$ ($\sim$ 20 - 50 GeV @ LHC)

to identify cluster of hadrons as jet

$\sigma_{njet} (E_{T_j} > E_T)$

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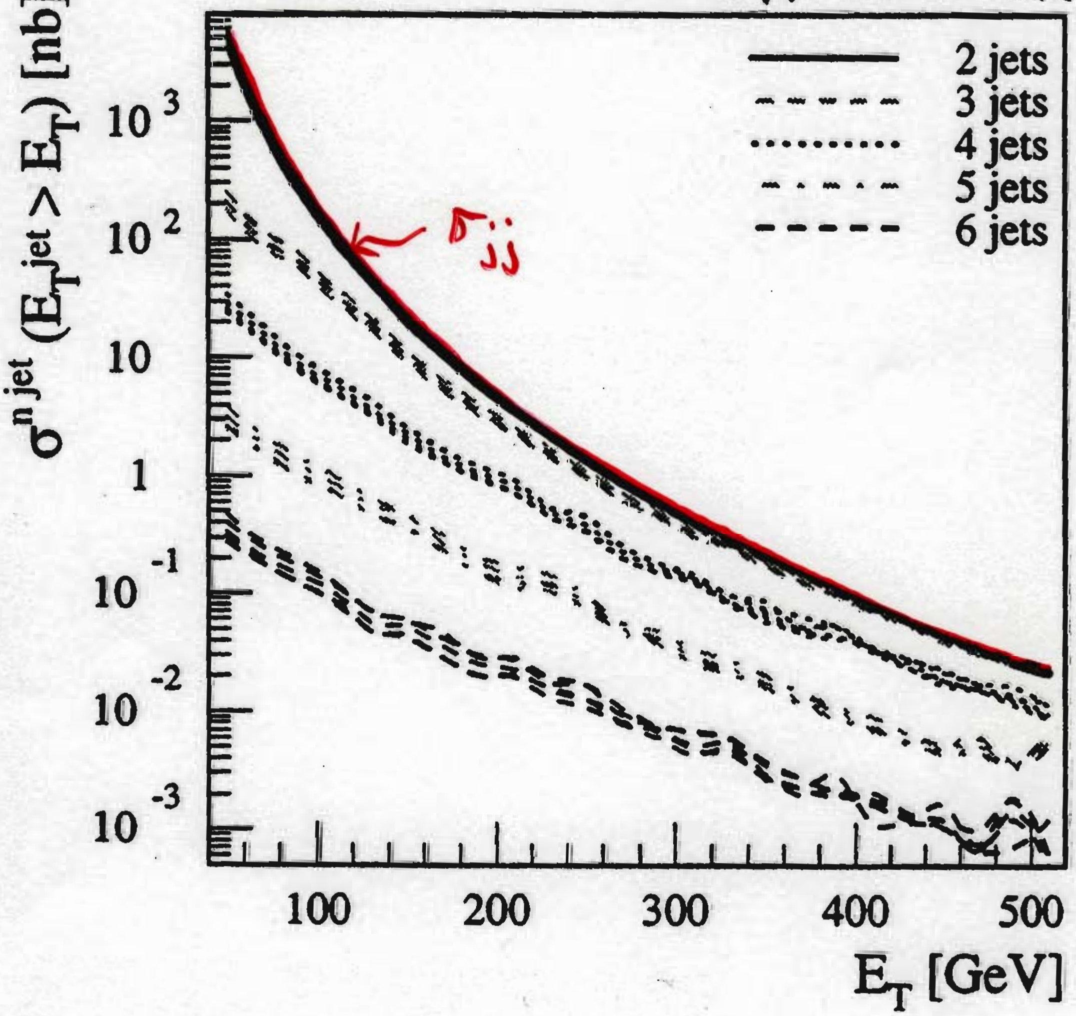
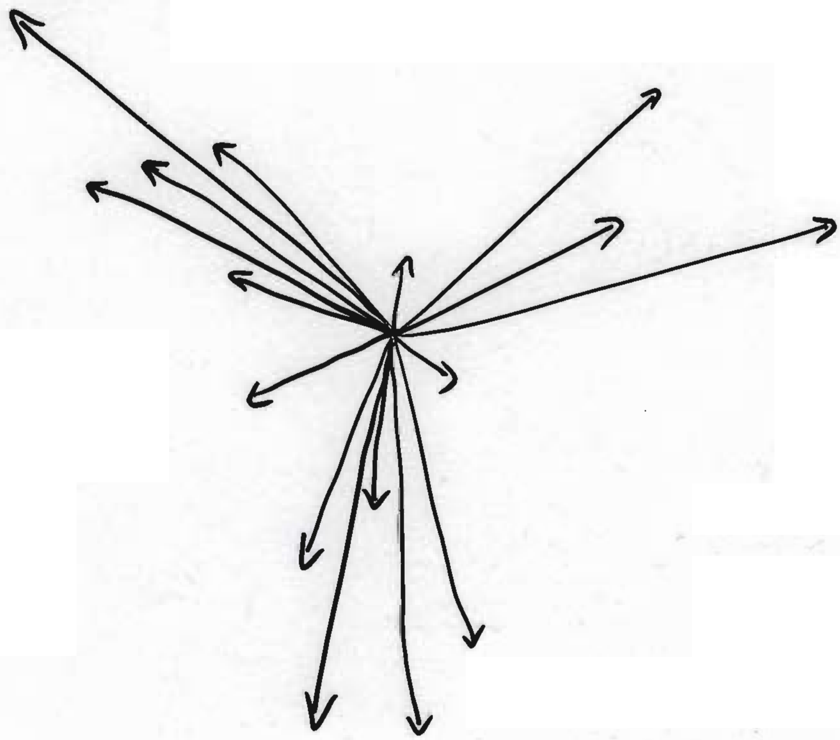


Jet algorithms (basic)

event with many hadrons / partons



problem: how to associate jets?

Form clusters of hadrons / partons

- decide how many clusters to form
  $\Rightarrow$ # of jets

- decide which particle to associate to which cluster
  $\Rightarrow$ momentum of jet

- cone scheme

criterion is separation in η-φ plane

$$\Delta R_{ij} = \sqrt{(\eta_i - \eta_j)^2 + (\varphi_i - \varphi_j)^2}$$

recombine partons / clusters with

$$\Delta R_{ij} < R_{cone} \qquad (0.4 \ldots 0.7)$$

call recombined clusters jets if

$$p_{Tj}^{lab} > p_{T,min} \qquad (20\text{ GeV or larger})$$

$$|\eta_j| < \eta_{max} \qquad (4.5 \ldots 5)$$

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significance of ΔR_{ij} for massless partons

$$p_1 = p_{T1}(\cosh\eta_1, \cos\varphi_1, \sin\varphi_1, \sinh\eta_1)$$

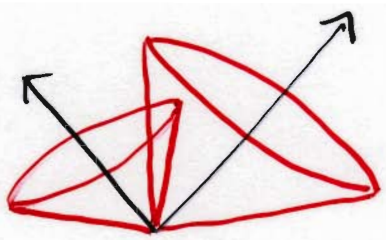
$$p_2 = p_{T2}(\cosh\eta_2, \cos\varphi_2, \sin\varphi_2, \sinh\eta_2)$$

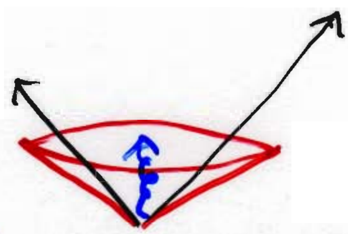
$$\Rightarrow p^2 = (p_1 + p_2)^2 = 4 p_{T1} p_{T2}\left[\sinh^2\frac{\eta_1 - \eta_2}{2} + \sin^2\frac{\varphi_1 - \varphi_2}{2}\right]$$

$$\approx p_{T1} p_{T2}\left[(\eta_1 - \eta_2)^2 + (\varphi_1 - \varphi_2)^2\right] = p_{T1} p_{T2} (\Delta R_{12})^2$$

Iterate cone position until momentum of particles inside cone points along cone axis

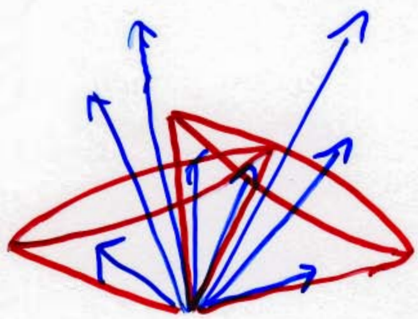
Problem 1: infrared safety





Solution: try all possible cone positions not just cones around "particles"

Problem 2: overlapping stable cones



solution:

ad hoc prescription for assigning particle momenta in overlap region to jets

- inclusive k_T-algorithm with R-separation

Ellis & Soper

successively merge softest partons to nearest neighbors in R_{ij}

- for each pair of "protojets" compare

$$d_{ij} = \min(E_{Ti}^2, E_{Tj}^2) \frac{(y_i - y_j)^2 + (\varphi_i - \varphi_j)^2}{R^2}$$

with

$$d_i = E_{Ti}^2$$

- If $\min\{d_{ij}\} < \min\{d_i\}$ merge i and j to new protojet.

- If $\min\{d_i\} < \min\{d_{ij}\}$ move cluster i to list of jets. Continue above with remaining protojets.

Impose E_T and η cuts on jets in final jet list.

R plays role of cone size.

Differences are important in practice

- NLO corrections to jet cross sections

- jet energy correction due to underlying event

- dijet invariant mass resolution

  e.g. $H \to b\bar{b}$

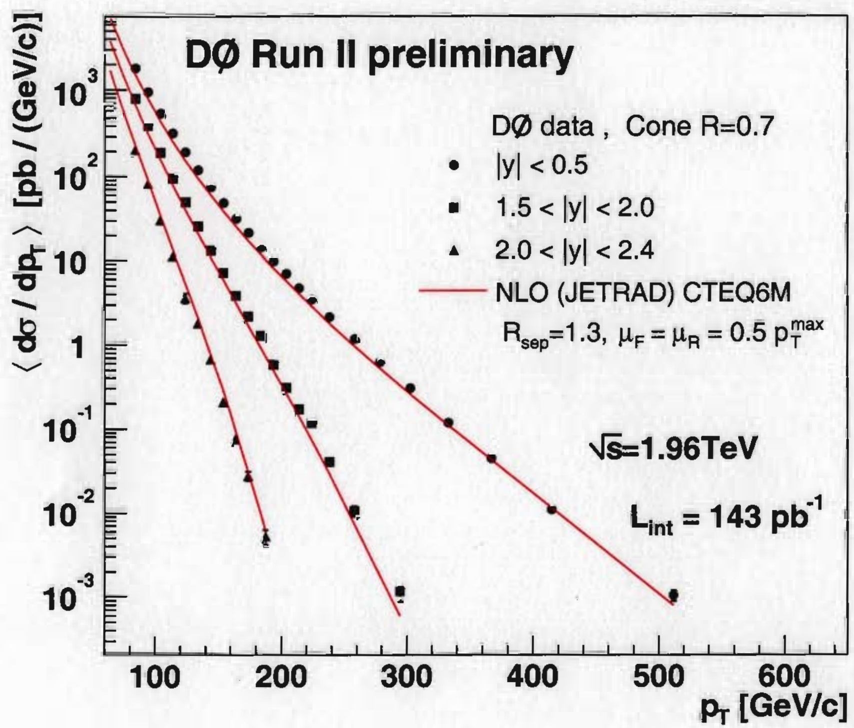
For details see e.g.

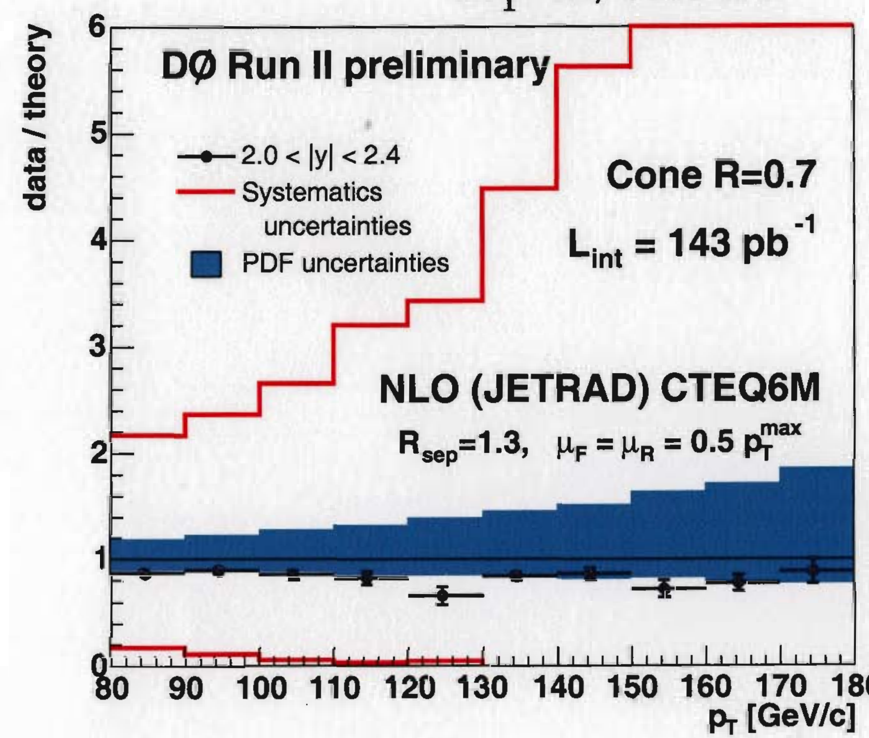
hep-ex/0005012 : Run II Jet Physics

$\to$ learn from Tevatron

Tevatron Inclusive Jet p_T Distributions

hep-ex/0409002





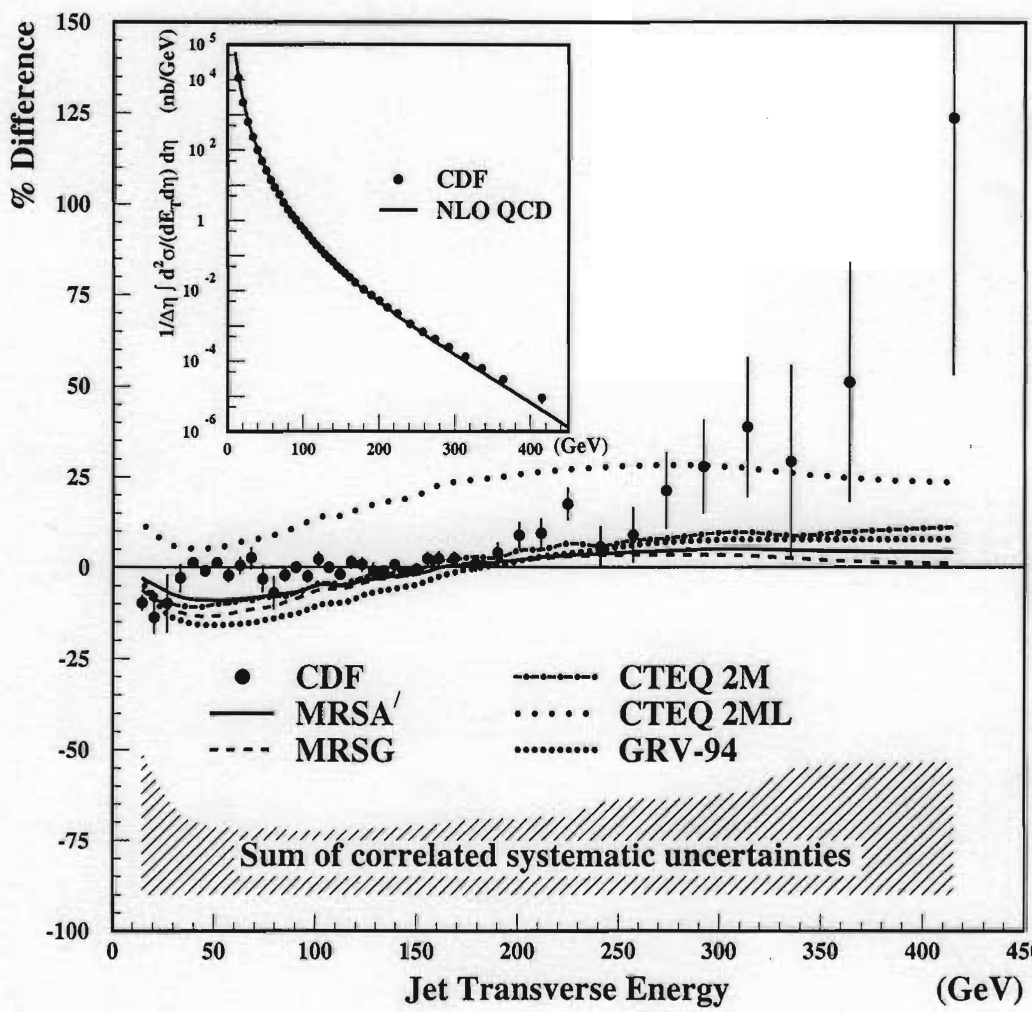
- Agreement with NLO QCD at 10–20% level over more than 6 orders of magnitude
- Steep p_T dependence $\Rightarrow$ jet rates depend totally on applied cuts:
  no *back-of-the-envelope* estimates

Historical interlude:

High E_T jet excess at Tevatron

New physics ?

Breakdown of perturbative QCD ???



CDF
PRL 77
(1996)
438

Figure 1: The percent difference between the CDF inclusive jet cross section (points) and a next-to-leading order (NLO) QCD prediction using MRSD0′ PDFs. The CDF data (points) are compared directly to the NLO QCD prediction (line) in the inset. The normalization shown is absolute. The hatched region at the bottom shows the quadratic sum of correlated systematic uncertainties. NLO QCD predictions using different PDFs are also compared with the one using MRSD0′.

Effect of contact interaction

$$\mathcal{L}_{eff} = \frac{4\pi}{\Lambda^2}\,\bar{q}(x)\gamma^\mu q(x)\,\bar{q}(x)\gamma_\mu q(x)$$

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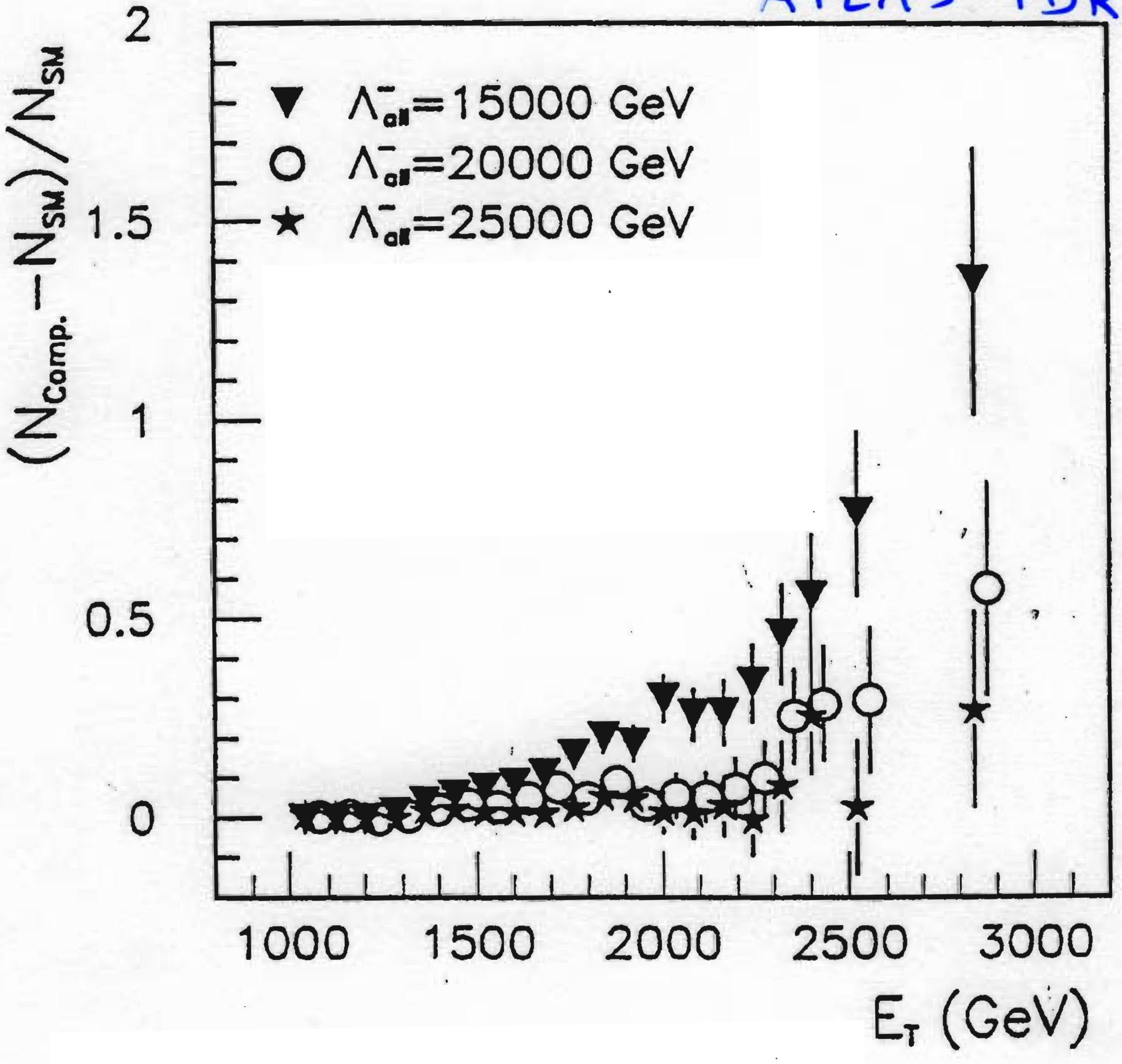


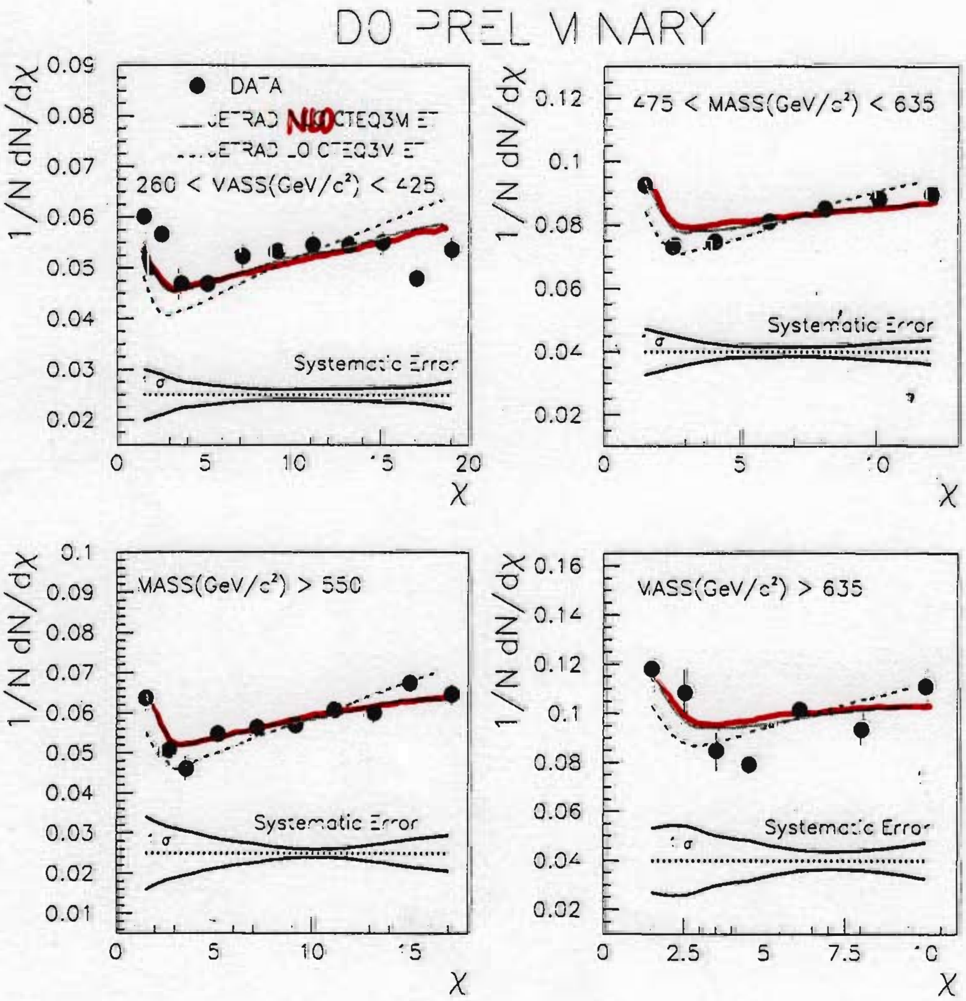
Exchange of heavy quanta leads to characteristic increase of jet cross section at high E_T

Angular distribution of dijets

$$\chi = \frac{1+\cos\theta^*}{1-\cos\theta^*} = e^{|\eta_1-\eta_2|} = e^{2\eta^*}$$

QCD $\frac{d\sigma}{d\chi}$ flat due to $\frac{1}{u}, \frac{1}{t}$ poles

Contact terms: $\frac{d\sigma}{d\cos\theta^*}$ flat $\Rightarrow$ falloff for large χ



⟶ = NLO QCD pred.

Solution: $g(x)$ larger than anticipated at $x \gtrsim 0.3$ $\Rightarrow$ additional parameters for high x tail of pdf's

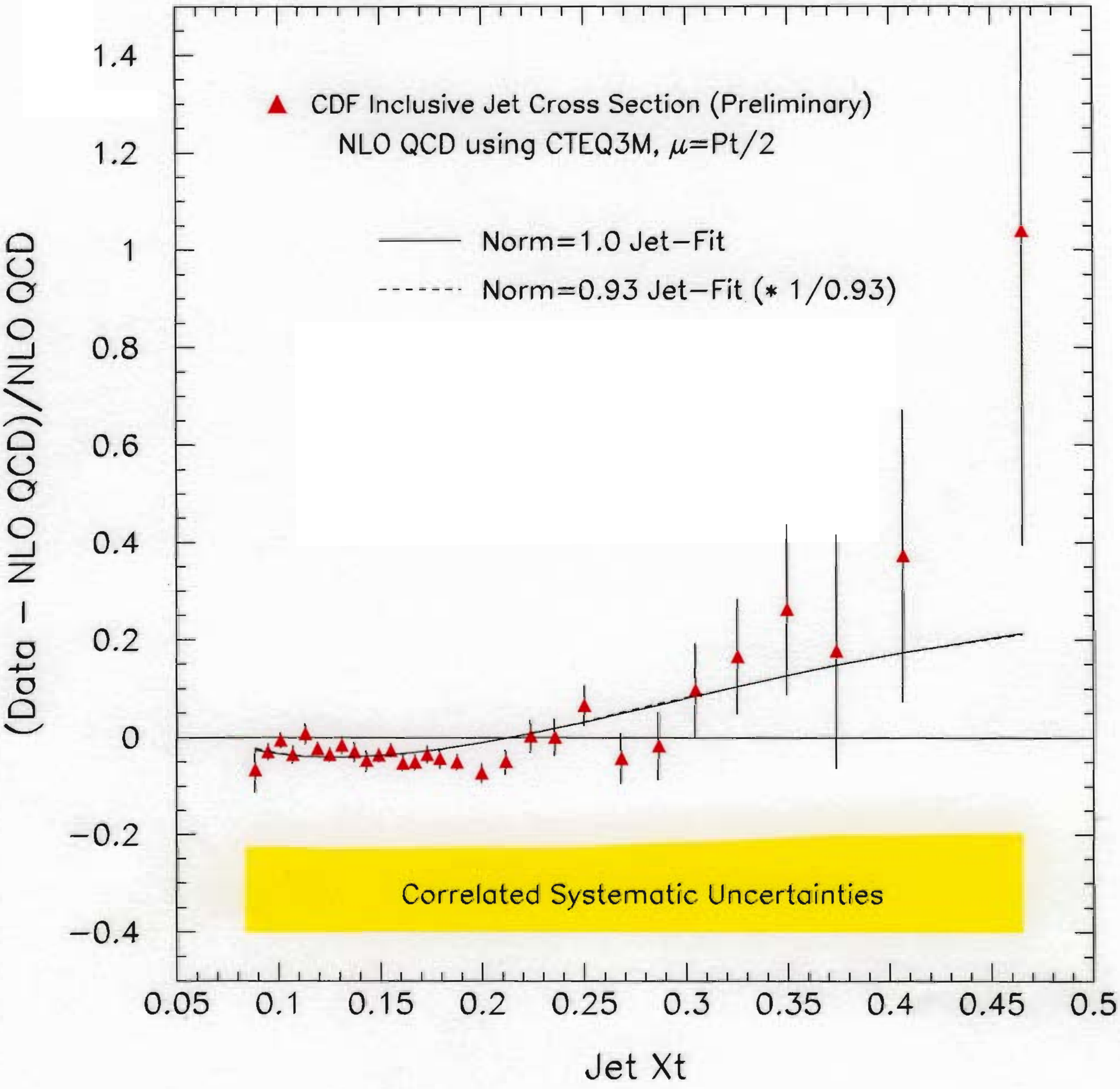


Figure 1: The preliminary CDF jet data is compared to a NLO QCD calculation using the conventional CTEQ3M parton distributions (points), and the new parton distributions fit to the jet data (solid and dashed lines that lie on top of each other).

25