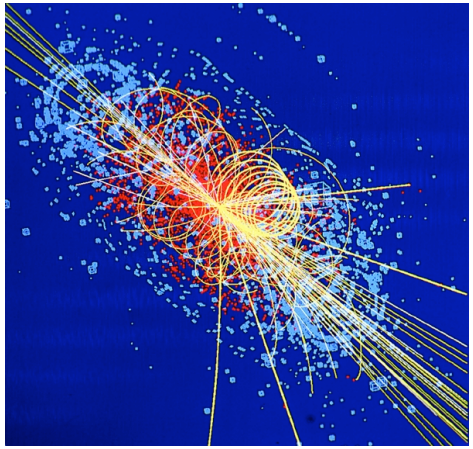
# QCD at Colliders
# Lecture 1



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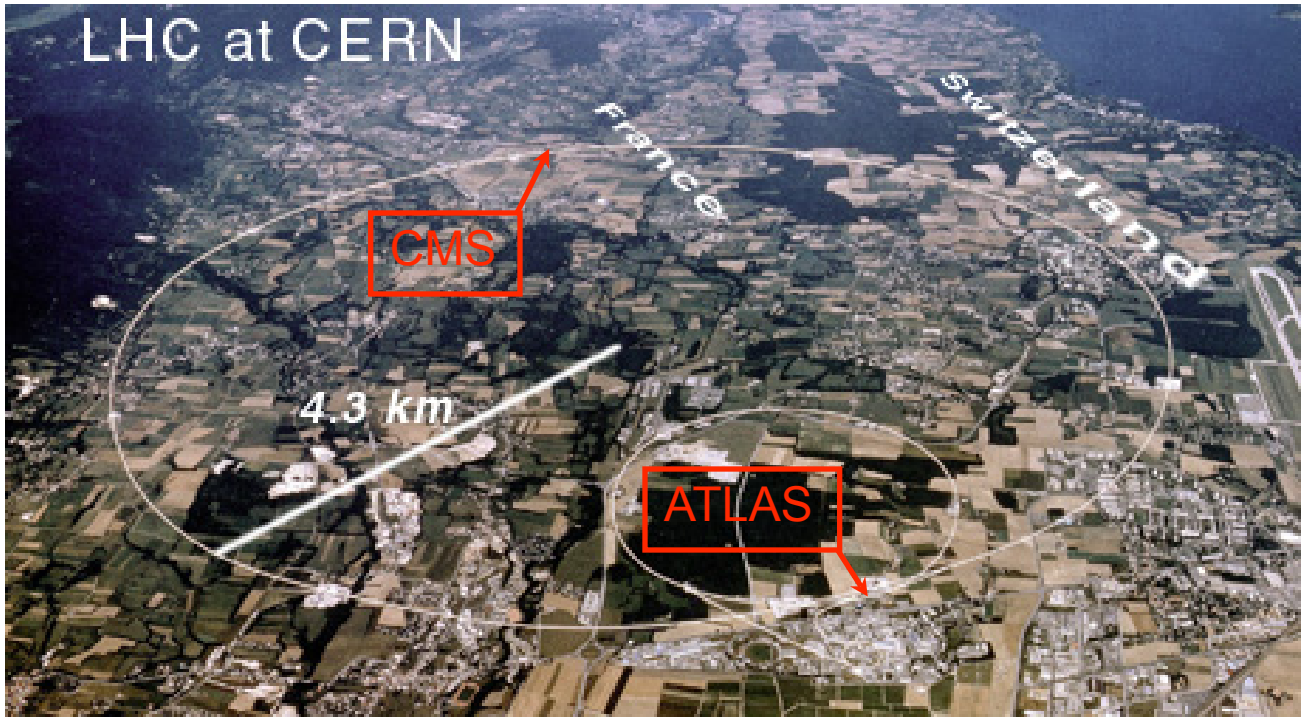
Prospects in Theoretical Particle Physics
Institute for Advanced Study, Princeton
July 19, 2005

# The Large Hadron Collider

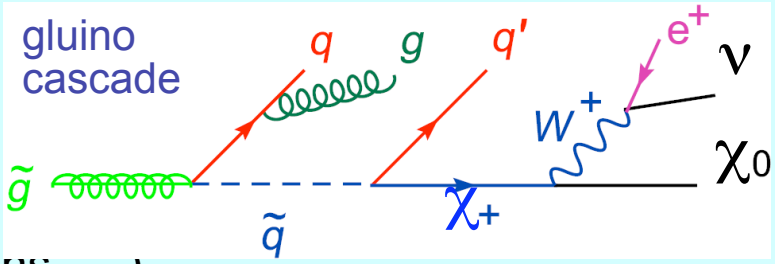



- Proton-proton collisions at **14 TeV** center-of-mass energy, 7 times greater than previous (Tevatron)
- Luminosity (collision rate) 10—100 times greater

- New window into electroweak-scale physics – opening 2007

# Signals and backgrounds

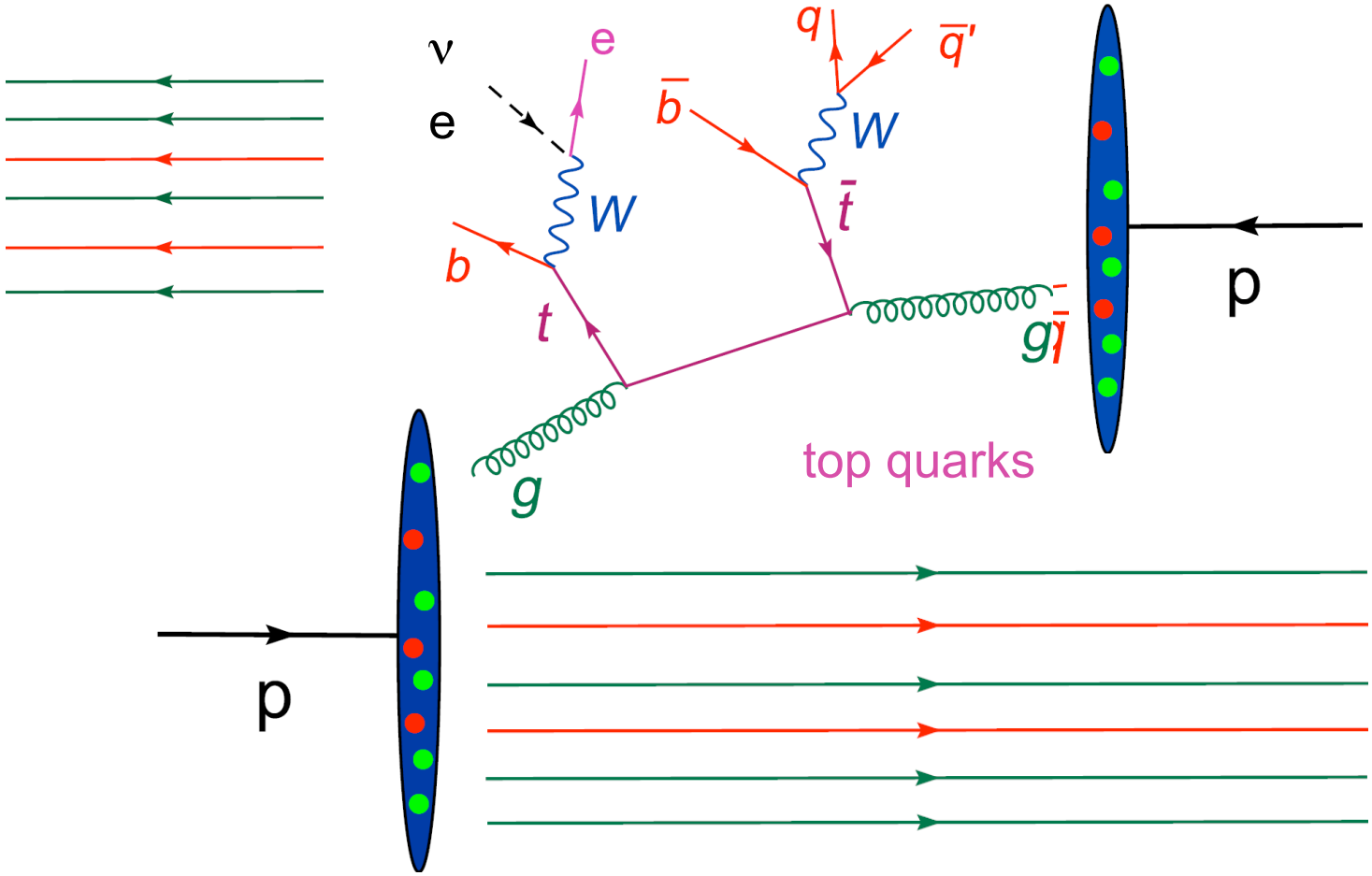
- Want to find new particles – whether from
  - supersymmetry
  - extra dimensions
  - new forces
- But new particles typically decay into old particles: quarks, gluons, charged leptons and neutrinos, photons, *W*s & *Z*s (which in turn decay to leptons, …)




- Kinematic signatures are not always clean (*e.g.* mass bumps) if neutrinos, or other escaping particles present

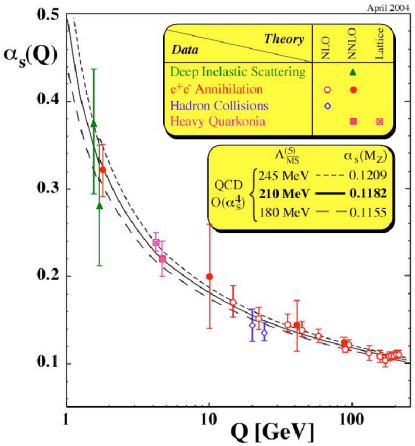
- Need to quantify the Standard Model backgrounds for a variety of multi-particle processes, to maximize potential for new physics discoveries

# What will the LHC see?



# QCD factorization & parton model

- Asymptotic freedom guarantees that at short distances (large transverse momenta), partons in the proton are almost free.
- They are sampled "one at a time" in hard collisions.
- Leads to QCD-improved parton model:




"suitable" final state

Parton distribution function:
prob. of finding parton *a* in proton 1, carrying fraction $x_1$ of its momentum

factorization scale ("arbitrary")

$$\sigma^{pp\to X}(s;\alpha_s,\mu_R,\mu_F) = \sum_{a,b}\int_0^1 dx_1\int_0^1 dx_2\, f_a(x_1,\alpha_s,\mu_F)\, f_b(x_2,\alpha_s,\mu_F) \times \hat{\sigma}^{ab\to X}(s x_1 x_2;\alpha_s,\mu_R,\mu_F)$$

Partonic cross section, computable in perturbative QCD

partonic CM energy$^2$

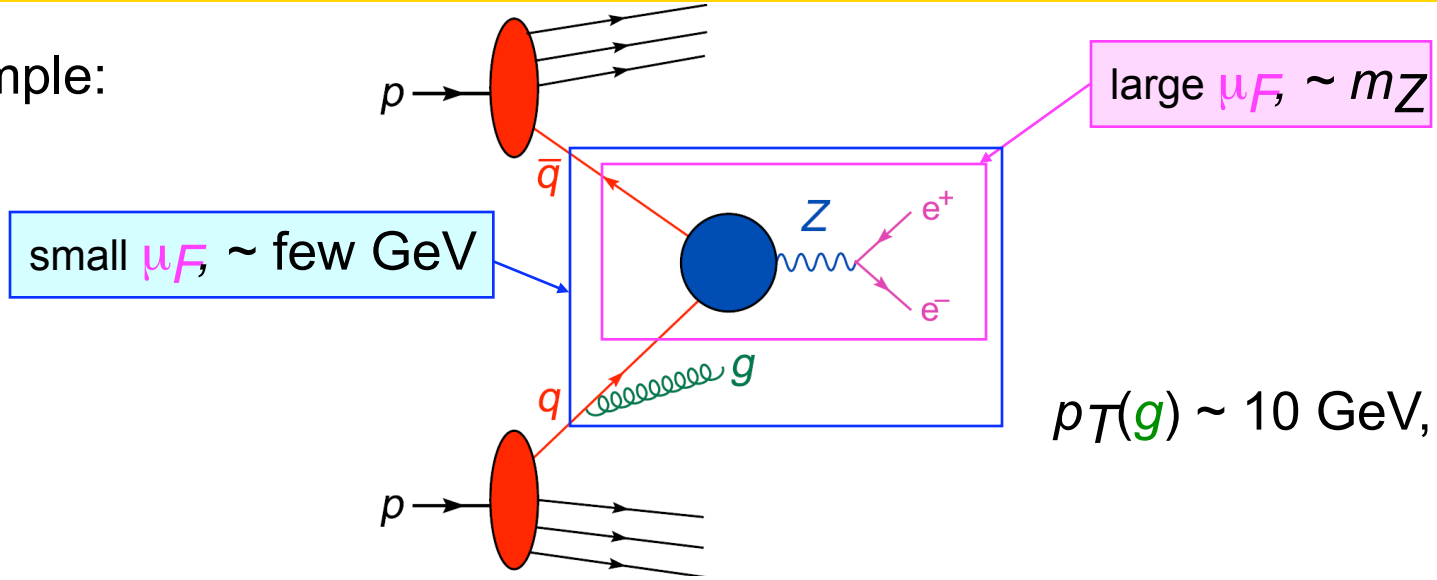
renormalization scale ("arbitrary")

# Parton evolution

- partons in the proton are not quite free
- distributions $f_a(x,\mu_F)$ evolve as scale $\mu_F$ at which they are resolved varies

$$\sigma^{pp\to X}(s;\alpha_s,\mu_R,\mu_F) = \sum_{a,b}\int_0^1 dx_1 \int_0^1 dx_2\; f_a(x_1,\alpha_s,\mu_F)\, f_b(x_2,\alpha_s,\mu_F) \times \hat{\sigma}^{ab\to X}(sx_1x_2;\alpha_s,\mu_R,\mu_F)$$

Example:




$p_T(g)$ ~ 10 GeV, say

# Parton evolution (cont.)

- parton distributions are nonperturbative
- must be measured experimentally
- experimental data at much lower $\mu_F^2$ than (100-1000 GeV)$^2$
- fortunately, evolution at $\mu_F$ > 1-2 GeV is perturbative
- DGLAP equation (return to later)

$$\mu^2 \frac{\partial}{\partial \mu^2} f_a(x,\mu) = \frac{\alpha_s(\mu)}{2\pi} \sum_b \int_x^1 \frac{d\xi}{\xi} P_{ab}(x/\xi, \alpha_s(\mu))\, f_b(\xi,\mu)$$

$$x = \frac{x}{\xi} \times \xi$$

$$P_{ab}(x,\alpha_s) = P_{ab}^{(0)}(x) + \frac{\alpha_s}{2\pi} P_{ab}^{(1)}(x) + \left(\frac{\alpha_s}{2\pi}\right)^2 P_{ab}^{(2)}(x) + \cdots$$

LO (1974) NLO (1980) NNLO (2004)

# Also expand partonic cross section:

$$\hat{\sigma}(\alpha_s, \mu_F, \mu_R) \;=\; [\alpha_s(\mu_R)]^{n_\alpha}\left[\hat{\sigma}^{(0)} + \frac{\alpha_s}{2\pi}\hat{\sigma}^{(1)}(\mu_F,\mu_R) + \left(\frac{\alpha_s}{2\pi}\right)^2 \hat{\sigma}^{(2)}(\mu_F,\mu_R) + \cdots\right]$$

LO NLO NNLO

**Problem:** Leading-order, tree-level predictions often only qualitative due to **poor convergence** of expansion in $\alpha_s(\mu)$

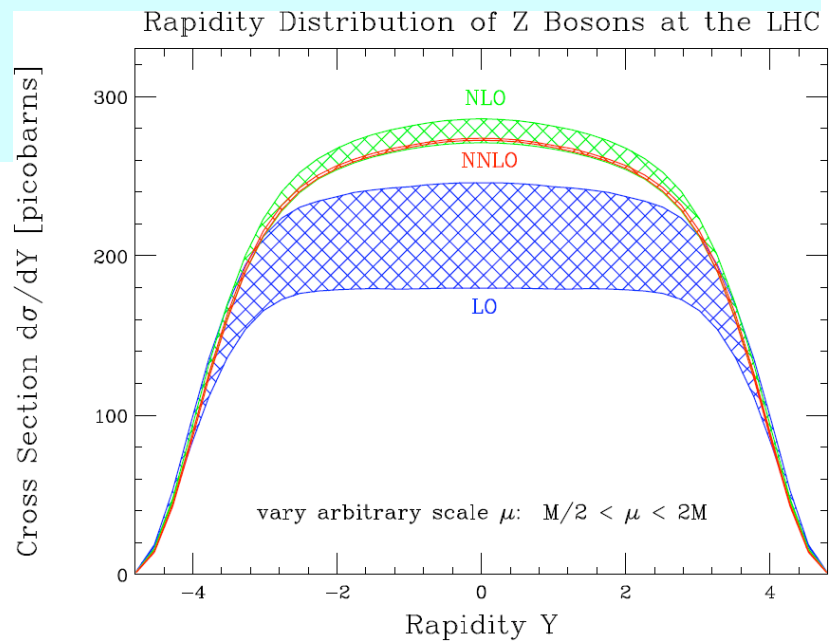
(setting $\mu_R = \mu_F = \mu$)

Example: $Z$ production at LHC. Predict distribution in rapidity

$$Y \;=\; \frac{1}{2}\ln\left(\frac{E+p_z}{E-p_z}\right)$$

$$\frac{d\sigma}{dY} \quad \text{has} \quad n_\alpha = 0$$

still 30% NLO corrections

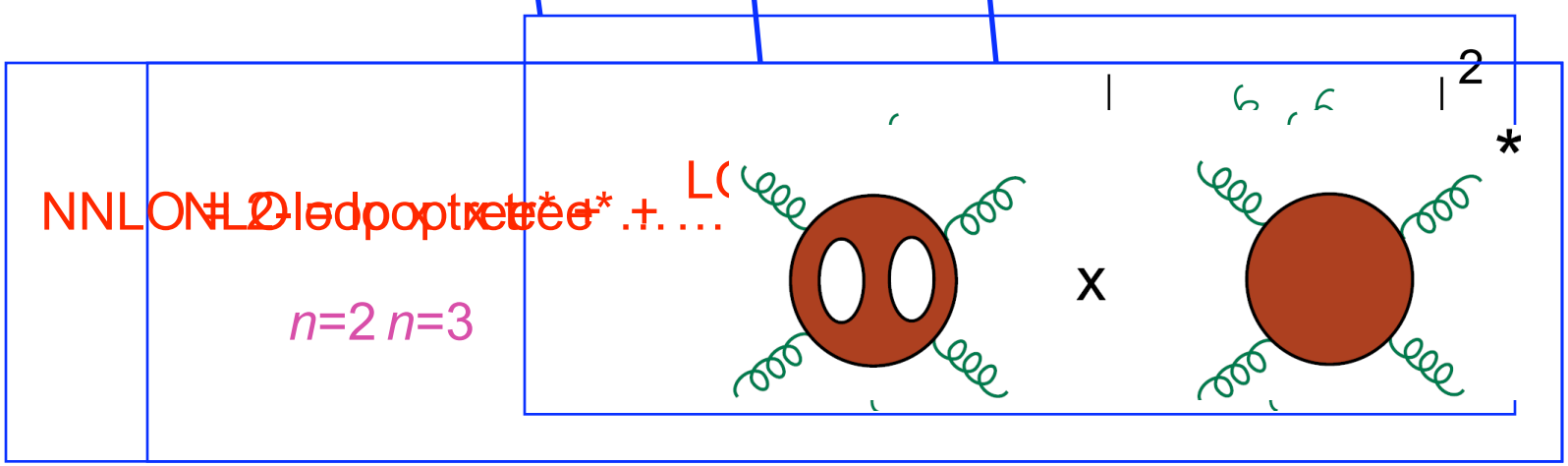


# What is the state of the art?

**Example of jet production**

(For W/Z/Higgs + jets, situation is similar or worse) if you count the W/Z/Higgs as a jet.)

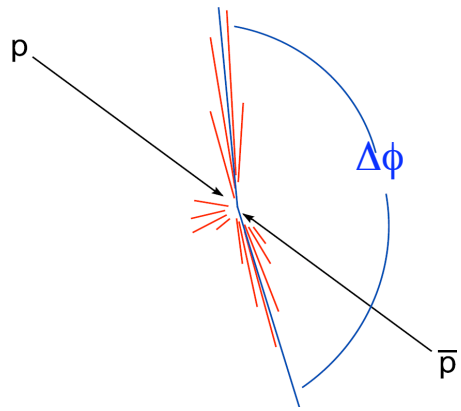
$$\sigma(\text{n jets}) \;=\; [\alpha_s(\mu)]^n \left[\hat{\sigma}^{(0)} + \frac{\alpha_s}{2\pi}\hat{\sigma}^{(1)}(\mu) + \left(\frac{\alpha_s}{2\pi}\right)^2 \hat{\sigma}^{(2)}(\mu) + \cdots\right]$$

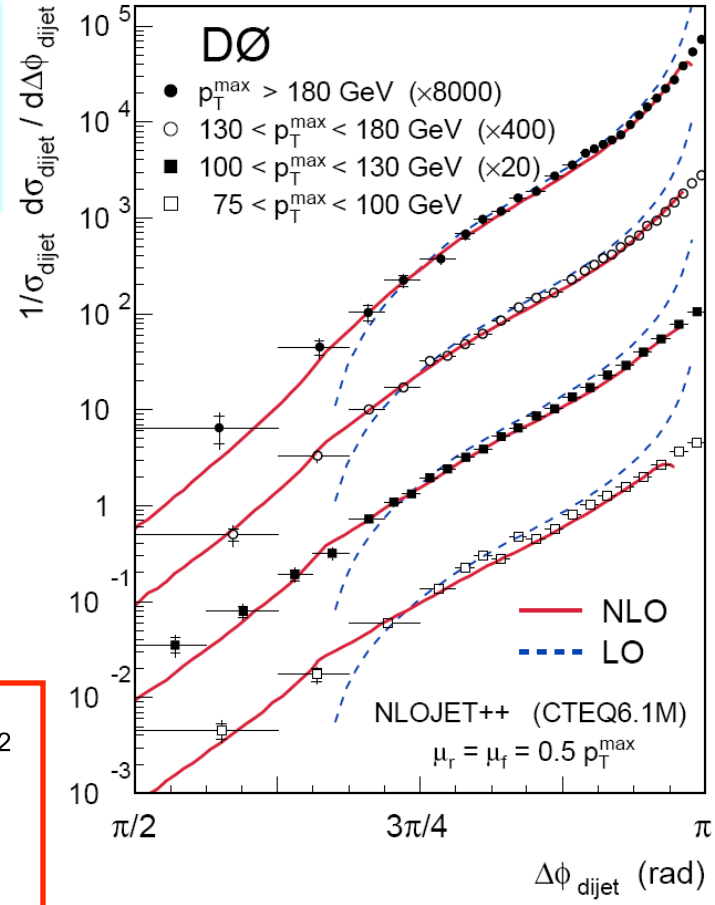


# NLO also improves distributions

Tevatron Run II example:

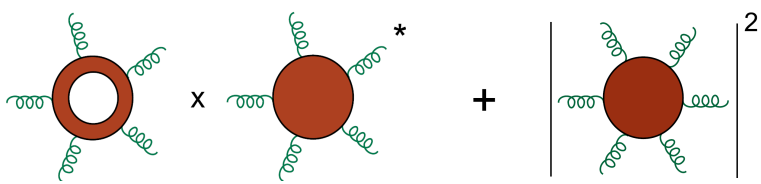
Azimuthal decorrelation of di-jets at D0 due to additional radiation




NLO

$n$=3



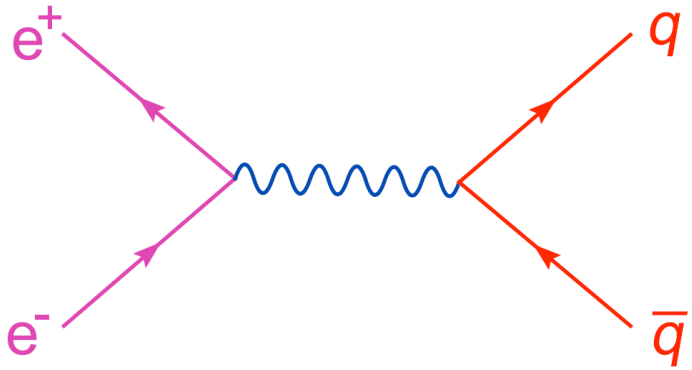
# Outline

1. Motivation
2. Helicity amplitudes and soft/collinear properties in spinor helicity formalism
3. Nature of IR cancellations
4. A NLO example, Drell-Yan (W/Z) production
5. General NLO structure
6. Why are corrections big? Soft gluon resummation.

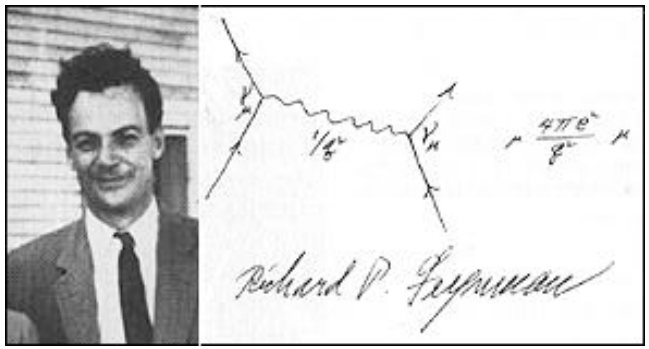
Thurs. afternoon: New (twistor-inspired) techniques for computing tree and loop amplitudes

# The most famous Feynman diagram

$e^+ e^-$ annihilation to jets




signed by the author



# Spinor interlude

Scattering amplitudes for massless plane waves of definite momentum:
Lorentz 4-vectors $k_i^\mu$ $\quad k_i^2=0$

Natural to use Lorentz-invariant products (invariant masses): $s_{ij} = 2k_i \cdot k_j = (k_i + k_j)^2$

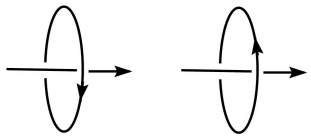
But for elementary particles with **spin** (*e.g.* all observed ones!) **there is a better way:**

**Take "square root"** of 4-vectors $k_i^\mu$ (spin 1)

use Dirac (Weyl) spinors $u_\pm(k_i)$ (spin 1/2)

right-handed: $(\lambda_i)_\alpha = u_+(k_i)$ left-handed: $(\tilde{\lambda}_i)_{\dot{\alpha}} = u_-(k_i)$

$q$,$g$,$\gamma$, all have 2 helicity states, $h = \pm$



# Spinor products

Instead of Lorentz products: $s_{ij} = 2k_i \cdot k_j = (k_i + k_j)^2$

Use spinor products: $\bar{u}_-(k_i)u_+(k_j) = \varepsilon^{\alpha\beta}(\lambda_i)_\alpha(\lambda_j)_\beta = \langle i\, j\rangle$

$$\bar{u}_+(k_i)u_-(k_j) = \varepsilon^{\dot\alpha\dot\beta}(\tilde\lambda_i)_{\dot\alpha}(\tilde\lambda_j)_{\dot\beta} = [i\, j]$$

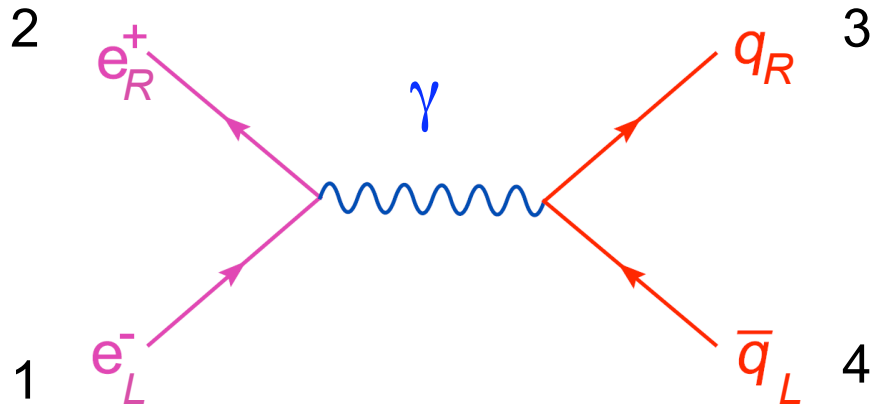
Identity $k_i^\mu(\sigma_\mu)_{\alpha\dot\alpha} = (\not{k}_i)_{\alpha\dot\alpha} = u_+(k_i)\bar{u}_+(k_i) = (\lambda_i)_\alpha(\tilde\lambda_i)_{\dot\alpha}$

$\Rightarrow$ These are **complex square roots** of Lorentz products:

$$\langle i\, j\rangle[j\, i] = \tfrac{1}{2}\mathrm{Tr}\left[\not{k}_i\ \not{k}_j\right] = 2k_i \cdot k_j = s_{ij}$$

$$\boxed{\langle i\, j\rangle = \sqrt{s_{ij}}\, e^{i\phi_{ij}} \qquad [j\, i] = \sqrt{s_{ij}}\, e^{-i\phi_{ij}}}$$

# That diagram again




add helicity information, numeric labels

$$\mathcal{A}_4 = 2ie^2 Q_e Q_q \delta^{\bar{\imath}_4}_{i_3} A_4$$

$$A_4 = \frac{1}{2\,s_{12}} \overline{v_-}(k_2)\gamma^\mu u_-(k_1)\ \overline{u_+}(k_3)\gamma_\mu v_+(k_4)$$

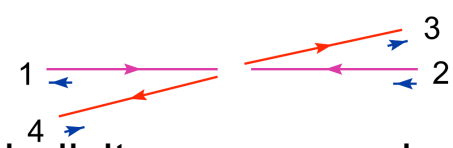
$$= \frac{1}{2\,s_{12}} (\sigma^\mu)_{\alpha\dot\alpha}(\lambda_2)^\alpha(\tilde\lambda_1)^{\dot\alpha}\ (\sigma_\mu)^{\dot\beta\beta}(\tilde\lambda_3)_{\dot\beta}(\lambda_4)_\beta$$

$$= \frac{1}{s_{12}} (\lambda_2)^\alpha(\tilde\lambda_1)^{\dot\alpha}(\lambda_4)_\alpha(\tilde\lambda_3)_{\dot\alpha}$$

Fierz identity

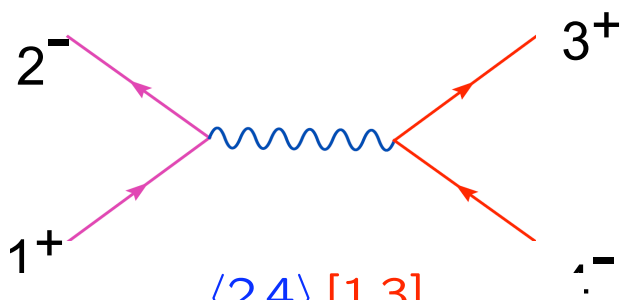
$$(\sigma^\mu)_{\alpha\dot\alpha}\,(\sigma_\mu)^{\dot\beta\beta} = 2\ \delta^\beta_\alpha\,\delta^{\dot\beta}_{\dot\alpha}$$

$$A_4 = \frac{\langle 2\,4\rangle\,[1\,3]}{s_{12}} = e^{i\phi}\frac{s_{13}}{s_{12}} = \frac{-e^{i\phi}}{2}(1-\cos\theta)$$




helicity suppressed as 1 || 3 or 2 || 4

# Sometimes useful to rewrite answer




Crossing symmetry more manifest
if we switch to all-outgoing helicity labels
(flip signs of incoming helicities)

$$A_4 = \frac{\langle 2\,4\rangle\,[1\,3]}{s_{12}}$$

$$= \frac{\langle 2\,4\rangle\,[1\,3]}{\langle 1\,2\rangle[2\,1]}\frac{\langle 1\,3\rangle}{\langle 1\,3\rangle}$$

$$= -\frac{\langle 2\,4\rangle\,[2\,4]\langle 2\,4\rangle}{\langle 1\,2\rangle[2\,4]\langle 4\,3\rangle}$$

$$\boxed{A_4 = \frac{\langle 2\,4\rangle^2}{\langle 1\,2\rangle\langle 3\,4\rangle}}$$

"holomorphic"

or $$\boxed{A_4 = \frac{[1\,3]^2}{[1\,2][3\,4]}}$$

"antiholomorphic"

**useful identities**

$$\langle i\,j\rangle = -\langle j\,i\rangle$$

$$[i\,j] = -[j\,i]$$

$$\langle i\,i\rangle = [i\,i] = 0$$

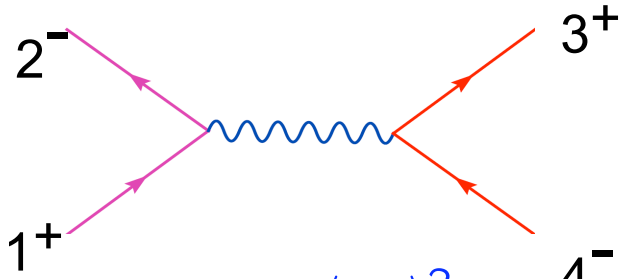
$$\langle i\,j\rangle\,[j\,i] = s_{ij}$$

$$\sum_{j=1}^{4} \langle i\,j\rangle\,[j\,k] = 0$$

$$s_{12} = s_{34}$$

$$s_{13} = s_{24}$$

# Symmetries for all other helicity config's

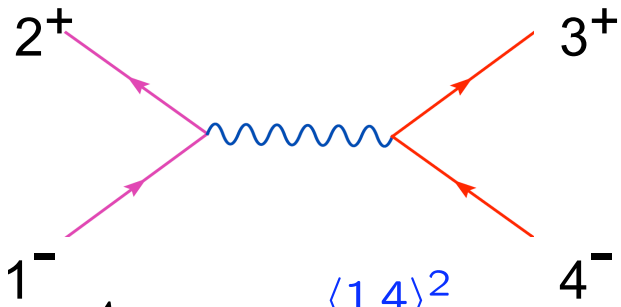



$$A_4 = \frac{\langle 2\,4\rangle^2}{\langle 1\,2\rangle\langle 3\,4\rangle}$$

C

$1 \leftrightarrow 2$

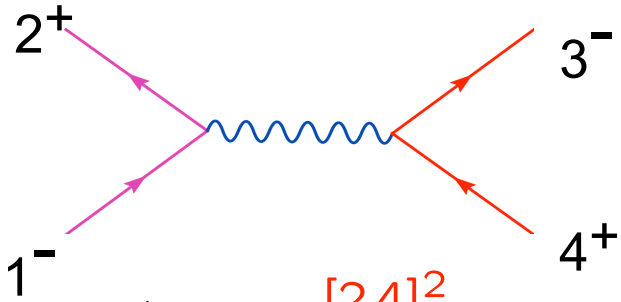
$\Longrightarrow$




$$A_4 = -\frac{\langle 1\,4\rangle^2}{\langle 1\,2\rangle\langle 3\,4\rangle}$$

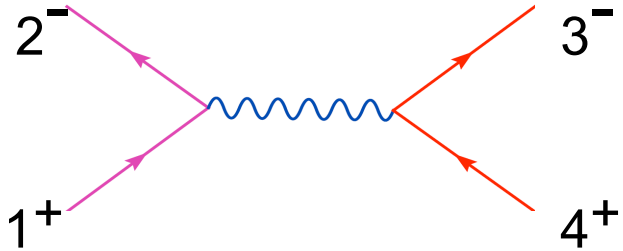
P $\Downarrow$ $\langle\,\rangle \leftrightarrow [\,]$

$\Downarrow$




$$A_4 = \frac{[2\,4]^2}{[1\,2][3\,4]}$$

$\Longrightarrow$




$$A_4 = -\frac{[1\,4]^2}{[1\,2][3\,4]}$$

# Unpolarized, helicity-summed cross sections

(the norm in QCD)

$$\frac{d\sigma(e^+e^- \to q\bar{q})}{d\cos\theta} \propto \sum_{\text{hel.}} |A_4|^2 = 2\left\{\left|\frac{\langle 2\,4\rangle^2}{\langle 1\,2\rangle\langle 3\,4\rangle}\right|^2 + \left|\frac{\langle 1\,4\rangle^2}{\langle 1\,2\rangle\langle 3\,4\rangle}\right|^2\right\}$$

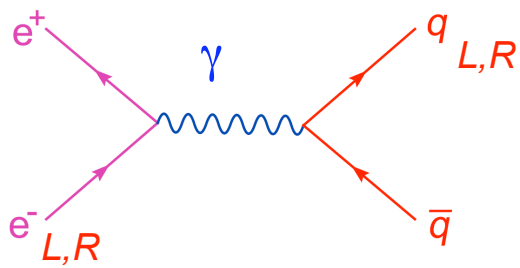
$$= 2\,\frac{s_{24}^2 + s_{14}^2}{s_{12}^2}$$

$$= \frac{1}{2}\left[(1-\cos\theta)^2 + (1+\cos\theta)^2\right]$$

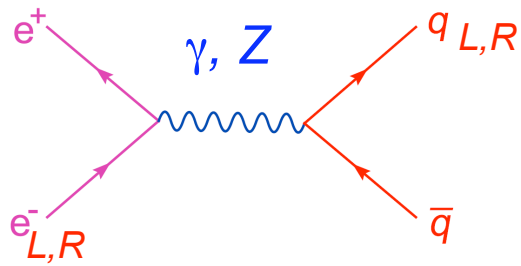
$$= 1 + \cos^2\theta$$

# Reweight helicity amplitudes for electroweak

For example, $Z$ exchange




$\Rightarrow$

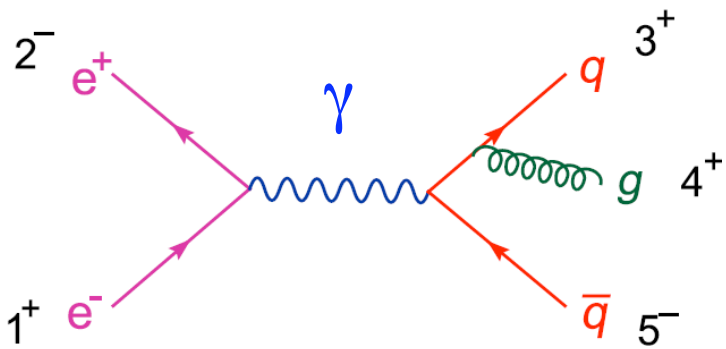



$$Q_e Q_q \quad \Rightarrow \quad Q_e Q_q + \frac{v^e_{L,R} v^q_{L,R}\ s}{s - M_Z^2 + i\Gamma_Z M_Z}$$

$$v_L^f = \frac{2I_3^f - 2Q_f \sin^2\theta_W}{\sin 2\theta_W} \qquad v_R^f = -\frac{2Q_f \sin^2\theta_W}{\sin 2\theta_W}$$

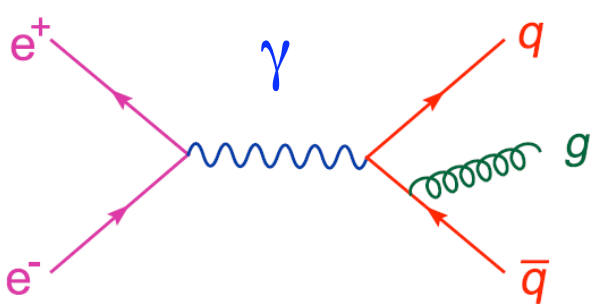
# Next most famous pair of Feynman diagrams

(to a higher-order QCD person)




$$\mathcal{A}_5 = 2ie^2 g Q_e Q_q (T^{a_4})^{\bar{\imath}_5}_{i_3} A_5$$

$$A_5 = \frac{\langle 2\,5\rangle}{s_{12}} \frac{\langle 1^+|(\not{k}_3 + \not{k}_4)\not{\epsilon}_4^+|3^-\rangle}{\sqrt{2}\, s_{34}} + \frac{[1\,3]}{s_{12}} \frac{\langle 2^-|(\not{k}_4 + \not{k}_5)\not{\epsilon}_4^+|5^+\rangle}{\sqrt{2}\, s_{45}}$$



# Next time

- $A_5$ and other multi-parton amplitudes, soft and collinear properties
- From splitting amplitudes to splitting probabilities (DGLAP)
- Implications of collinear structure (infrared safety)
- IR cancellations in $e^+e^-$