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STRING THEORY ...

NEW FRAMEWORK FOR PHYSICS THAT GOES BEYOND QUANTUM FIELD THEORY

MAKES IT POSSIBLE TO INCLUDE GRAVITY + QUANTUM THEORY IN A CONSISTENT WAY, NEW FRAMEWORK FOR UNIFYING FORCES OF NATURE

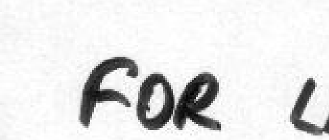
STRING THEORY IS BASED ON A NEW KIND OF GEOMETRY, GOING BEYOND STANDARD DIFFERENTIAL GEOMETRY, THAT WE DO NOT YET UNDERSTAND WELL.

SOMEHOW, IT YIELDS MANY NEW INSIGHTS ABOUT PHYSICAL THEORIES OF AN ESTABLISHED KIND- QUANTUM GAUGE THEORIES.

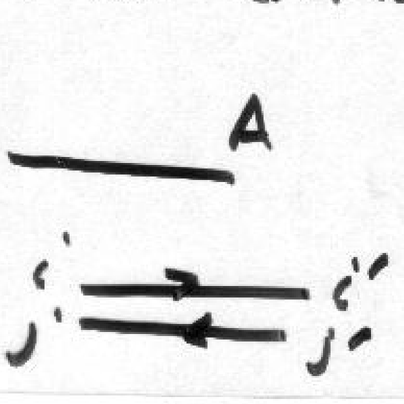
A MAIN THEME IN THIS SCHOOL WILL BE GAUGE/GRAVITY CORRESPONDENCE – USE OF STRING THEORY TO DEDUCE GAUGE THEORY RESULTS FROM GRAVITY, OR VICE-VERSA

THIS WILL BE A THEME OF MANY OF THE LECTURERS.

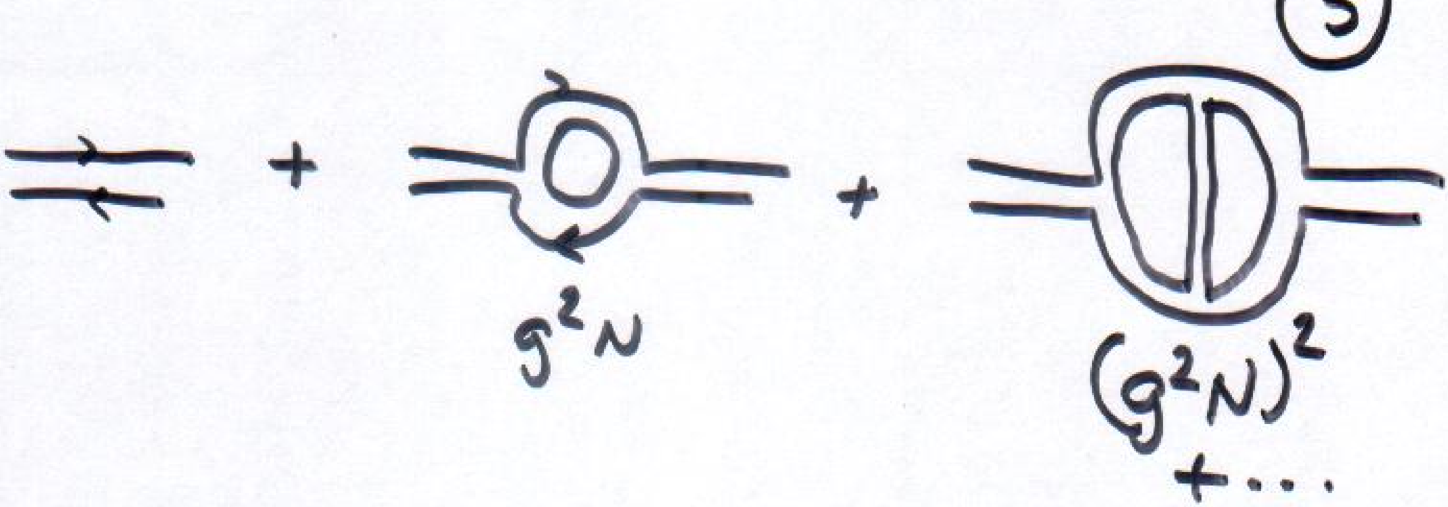
THE FIRST HINT OF GAUGE/STRING OR IN MODERN LANGUAGE GAUGE/GRAVITY DUALITY GOES BACK AT LEAST TO 'T HOOFT (1974). AIMING TO EXPLAIN THE UNRESOLVED ASPECTS OF QCD – QUARK CONFINEMENT, CHIRAL SYMMETRY BREAKING, MASS GENERATION – 't HOOFT CONSIDERED SU(N) GAUGE THEORY FOR LARGE N, WITH $g \to 0$



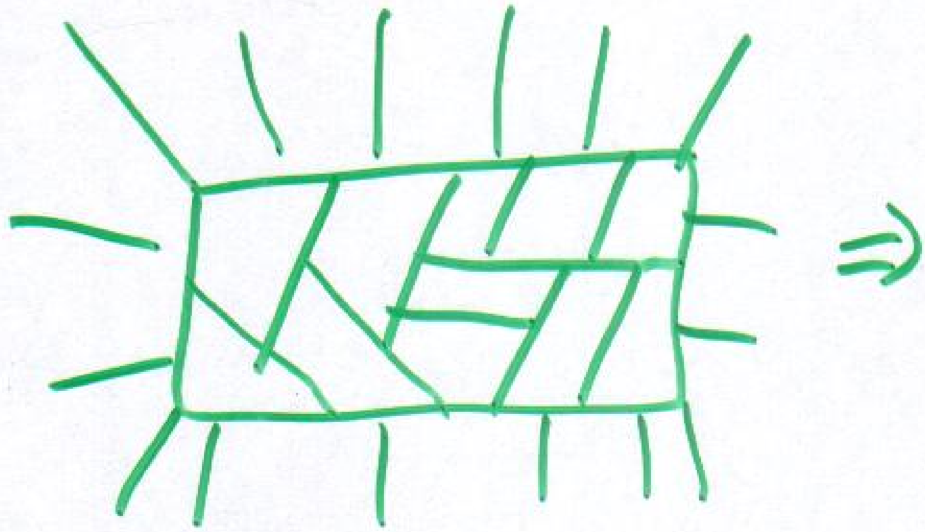
$g^2 N$ FIXED $\quad A^i_j$

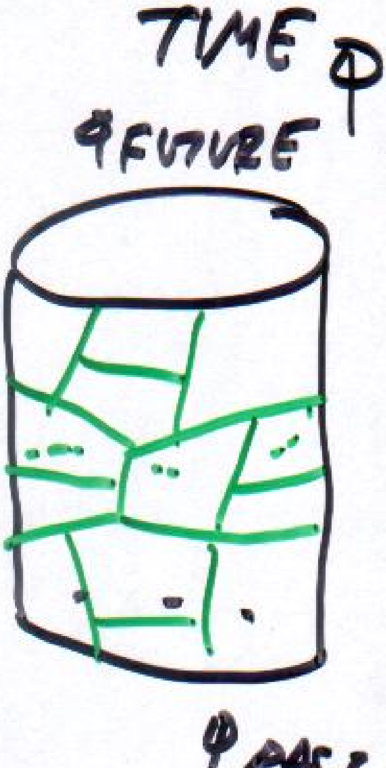


$\langle A^i_j \, A^{j'}_{i'} \rangle = \delta^i_{i'} \, \delta^{j'}_j \qquad i,j = 1 .. N \quad N \times N$



ALL FEYNMAN DIAGRAMS
THAT ARE "PLANAR"





SUGGESTS: NONPERTURBATIVE
EFFECTS FILL IN THE HOLES, QCD
= A STRING THEORY WITH $1/N$ AS THE
STRING COUPLING CONSTANT

AS THIS SUGGESTS, ONE GOAL OF WORK ON GAUGE/STRING OR GAUGE/GRAVITY DUALITY IS TO UNDERSTAND NON PERTURBATIVE ASPECTS OF GAUGE THEORY THAT ARE NOT YET WELL UNDERSTOOD.

FIRST REAL BREAKTHROUGH TOOK 23 YEARS:

AdS/CFT CORRESPONDENCE
(MALDACENA)

See lectures by KIRITSIS, KLEBANOV, SKENDERIS

ADS/CFT AND ITS EXTENSIONS MAKE IT CLEAR THAT GAUGE/STRING DUALITY IS CORRECT AND IS USEFUL AT LEAST FOR LARGE g^2N.

BUT THAT ISN'T ENOUGH: QCD IS ASYMPTOTICALLY FREE, SO $g^2N \ll 1$ AT SMALL DISTANCES.

TO MAKE GAUGE/STRING DUALITY USEFUL FOR QCD, WE NEED TO UNDERSTAND IT FOR ALL g^2N.

THAT WILL BE THE GOAL OF

MY LECTURES:

FIND A STRING THEORY CONSTRUCTION

THAT IS RELEVANT TO FOUR-DIMENSIONAL

GAUGE THEORY AT SMALL g^2N,

i.e.

INTERPRET PERTURBATIVE

GAUGE THEORY IN FOUR

DIMENSIONS AS A STRING

THEORY

I CAN'T PROMISE THAT WHAT I'LL EXPLAIN WILL TURN OUT TO BE USEFUL IN A STRING DESCRIPTION OF QCD, BUT AT LEAST I'LL TELL YOU INTERESTING THINGS ABOUT PERTURBATIVE GAUGE THEORY!

START WITH THE FACT THAT

$$SO(3,1)_{\mathbb{C}} \simeq SL(2)_{\mathbb{C}} \times SL(2)_{\mathbb{C}}$$

WITH

$SO(4) \cong SU(2) \times SU(2)$

$$\text{VECTOR} \simeq \left(\tfrac{1}{2}, \tfrac{1}{2}\right)$$

SO A VECTOR p_μ CAN

BE EXPRESSED AS A BI-SPINOR

$p_{a\dot{a}}$ $\quad a = 1,2 \quad \dot{a} = 1,2 \quad$ BY

$$p_{a\dot{a}} = \sigma^{\mu}_{a\dot{a}} \, p_\mu \,, \quad \text{WHERE } \sigma^{\mu}$$

= CHIRAL Γ MATRICES

GAMMA

$$\sigma^\mu = (1, \vec{\sigma})$$

i.e.

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix}$$

IN A CHIRAL BASIS $\begin{pmatrix} \lambda_a \\ \tilde{\lambda}_{\dot{a}} \end{pmatrix}$

IF

$$p_{a\dot{a}} = \sigma^\mu_{a\dot{a}} \, p_\mu = p_0 + \vec{\sigma} \cdot \vec{p}$$

THEN

$$\det p_{a\dot{a}} = p_\mu p^\mu$$

$$\begin{pmatrix} p_0 + p_3 & 0 \\ 0 & p_0 - p_3 \end{pmatrix}$$

$$\det = p_0^2 - p_3^2 = p_\mu p^\mu$$

SO p_μ IS LIGHTLIKE IF AND ONLY IF $\det p_{a\dot{a}} = 0$,

$\Longleftrightarrow$ $p_{a\dot{a}}$ HAS RANK < 2

$\Longleftrightarrow$ RANK $(p_{a\dot{a}}) \leq 1$

$\Longleftrightarrow$

$$p_{a\dot{a}} = \lambda_a \tilde{\lambda}_{\dot{a}}$$

FOR SOME λ_a, $\tilde{\lambda}_{\dot{a}}$

$\lambda, \tilde{\lambda}$ ARE UNIQUE UP TO

$$(\lambda, \tilde{\lambda}) \to (t\lambda, t^{-1}\tilde{\lambda})$$

$$t \in \mathbb{C}^*$$

FOR A CHIRAL SPINOR SUCH AS λ^a, THERE IS NO INVARIANT "METRIC" δ_{ab} TO RAISE AND LOWER INDICES, BUT THERE IS AN INVARIANT ANTISYMMETRIC TENSOR

$$\epsilon_{ab} = -\epsilon_{ba} \qquad a, b = 1, 2$$

$$\epsilon_{12} = +1, \qquad \epsilon^{ab}\epsilon_{bc} = \delta^a{}_c$$

WE USE ϵ TO RAISE AND LOWER INDICES

$$\lambda_a = \epsilon_{ab}\lambda^b \qquad \lambda^b = \epsilon^{bc}\lambda_c$$

GIVEN TWO SPINORS λ, λ'

OF THE SAME CHIRALITY

WE DEFINE AN ANTISYMMETRIC

INNER PRODUCT

$$\langle \lambda, \lambda' \rangle = \epsilon_{ab} \lambda^a \lambda'^b$$

SO $\langle \lambda, \lambda' \rangle = - \langle \lambda', \lambda \rangle$

$\langle \lambda, \lambda \rangle = 0$

LIKEWISE FOR OPPOSITE CHIRALITY SPINORS $\tilde{\lambda}^{\dot{a}}$ WE HAVE AN INVARIANT ANTISYMMETRIC TENSOR $\epsilon_{\dot{a}\dot{b}} = -\epsilon_{\dot{b}\dot{a}}$ AND WE SET

$$\tilde{\lambda}_{\dot{a}} = \epsilon_{\dot{a}\dot{b}} \tilde{\lambda}^{\dot{b}}, \qquad \tilde{\lambda}^{\dot{a}} = \epsilon^{\dot{a}\dot{b}} \tilde{\lambda}_{\dot{b}}$$

AND GIVEN $\tilde{\lambda}^{\dot{a}}, \tilde{\lambda}'^{\dot{b}}$ WE SET

$$[\tilde{\lambda}, \tilde{\lambda}'] = \epsilon_{\dot{a}\dot{b}} \tilde{\lambda}^{\dot{a}} \tilde{\lambda}'^{\dot{b}}$$

IF p, q ARE TWO LIGHTLIKE VECTORS, WE SET

$$p_{a\dot{a}} = \lambda_a \tilde{\lambda}_{\dot{a}}$$

$$q_{a\dot{a}} = \lambda'_a \tilde{\lambda}'_{\dot{a}}$$

AND THEN WE HAVE

$$2 p \cdot q = \langle \lambda, \lambda' \rangle [\tilde{\lambda}, \tilde{\lambda}']$$

(2 = AN EXERCISE)

WHEN WE WRITE

$$p_{a\dot{a}} = \lambda_a \tilde{\lambda}_{\dot{a}}$$

λ AND $\tilde{\lambda}$ HAVE A SIMPLE

PHYSICAL MEANING:

λ IS THE WAVEFUNCTION OF

A MASSLESS PARTICLE OF

HELICITY $-\frac{1}{2}$ $\{\tilde{\lambda}$ OF HELICITY $+\frac{1}{2}\}$

TO SEE THIS, WE JUST

WRITE THE DIRAC EQN FOR

A PARTICLE OF HELICITY $-\frac{1}{2}$

$0 = \sigma^{\mu}_{a\dot{a}} \partial_\mu \psi^a$ $\sigma^{\mu}_{a\dot{a}} \frac{\partial}{\partial x^\mu} = \frac{\partial}{\partial x^{a\dot{a}}}$ 18

$$\frac{\partial}{\partial x^{a\dot{a}}} \psi^a = 0$$

IF $\psi^a(x) = w^a \exp i\, x_{a\dot{a}} p^{a\dot{a}}$

WE GET

$$i\, p_{a\dot{a}} w^a = 0 \Big/ \lambda_a w^a \cdot \tilde{\lambda}_{\dot{a}} = 0$$

WITH $p_{a\dot{a}} = \lambda_a \tilde{\lambda}_{\dot{a}}$

WE GET $\lambda_a w^a = 0$

i.e. $w^a = \text{const} \cdot \lambda^a$

$$\left(\lambda_a \lambda^a = \varepsilon_{ab} \lambda^b \lambda^a = 0\right)$$

OBVIOUSLY, IN A SIMILAR FASHION, $\tilde{\lambda}$ IS A WAVEFUNCTION FOR HELICITY $+\frac{1}{2}$

NOW I WANT TO EXPLAIN THE ANALOG FOR HELICITY ± 1:

USUALLY, WE DESCRIBE A MASSLESS PHOTON BY A POLARIZATION VECTOR ϵ_μ – OR $\epsilon_{a\dot{a}}$ –

$$\epsilon_{a\dot{a}} = \sigma^{\mu}_{a\dot{a}}\, \epsilon_\mu$$

$$p^\mu \epsilon_\mu = 0$$ 20

WITH

$$\epsilon_\mu \to \epsilon_\mu + p_\mu w$$

* CONSTRAINT

$$p^{a\dot{a}} \epsilon_{a\dot{a}} = 0$$

* GAUGE INVARIANCE

$$\epsilon_{a\dot{a}} \to \epsilon_{a\dot{a}} + p_{a\dot{a}} w$$

FOR ANY w

I CLAIM WE CAN TAKE THE POLARIZATION VECTORS TO BE ...

HELICITY $+1$

$$\epsilon^{+}_{a\dot{a}} = \frac{\mu_a \tilde{\lambda}_{\dot{a}}}{\langle \mu, \lambda \rangle}$$

WHERE μ IS ANY SPINOR NOT A MULTIPLE OF λ

HELICITY -1 SIMILARLY

$$\epsilon^{-}_{a\dot{a}} = \frac{\lambda_a \tilde{\mu}_{\dot{a}}}{[\tilde{\lambda}, \tilde{\mu}]}$$

WHERE $\tilde{\mu} \neq \tilde{\lambda}$

CLEARLY IT SUFFICES TO

EXPLAIN THE CASE OF HELICITY +1

$p^{a\dot{a}}\epsilon^{+}_{a\dot{a}}=0$ IS CLEAR

WITH $\epsilon^{+}_{a\dot{a}} = \dfrac{\mu_a \tilde{\lambda}_{\dot{a}}}{[\mu, \lambda]}$

SINCE $p^{a\dot{a}} = \lambda^a \tilde{\lambda}^{\dot{a}}$

AND $\tilde{\lambda}^{\dot{a}}\tilde{\lambda}_{\dot{a}} = 0$

MOREOVER ϵ^{+} IS INDEPENDENT

OF THE CHOICE OF μ UP

TO A GAUGE TRANSFORMATION

SINCE μ LIVES IN A TWO-DIMENSIONAL SPACE, ANY CHANGE IN μ IS

$$\delta\mu = \alpha\mu + \beta\lambda \quad \text{FOR SOME } \alpha, \beta$$

AND

$$\epsilon^{+}_{a\dot{a}} = \frac{\mu_a \tilde{\lambda}_{\dot{a}}}{\langle \mu, \lambda \rangle}$$

IS INVARIANT TO THE α TERM, AND CHANGES UNDER THE β TERM BY A GAUGE TRANSFORMATION

$$\delta\epsilon^{+}_{a\dot{a}} \sim \lambda_a \tilde{\lambda}_{\dot{a}} = p_{a\dot{a}}$$

SO WE HAVE SHOWN THAT

UP TO A GAUGE TRANSFORMATION

ϵ^{+} IS UNIQUELY DETERMINED BY

THE CHOICE OF λ

(OR EQUIVALENTLY THE CHOICE OF

$\tilde{\lambda}$ AS $p_{a\dot{a}} = \lambda_a \tilde{\lambda}_{\dot{a}}$)

UNDER $(\lambda, \tilde{\lambda}) \to (t\lambda, t^{-1}\tilde{\lambda})$

WE HAVE

$$\epsilon^{-} \to t^{+2}\epsilon^{-}, \quad \epsilon^{+} \to t^{-2}\epsilon^{+}$$

TO SHOW THAT ϵ^+ DESCRIBES

HELICITY $+1$, WE MUST SHOW

THAT THE CORRESPONDING

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

IS SELFDUAL; I LEAVE THIS

AS AN EXERCISE.

"Pert G. Th as a Str Th

In Twistor Space"

SO FAR, WE'VE SEEN THAT

THE WAVE FUNCTION OF A

MASSLESS PARTICLE OF HELICITY h

SCALES UNDER $(\lambda, \tilde{\lambda}) \to (t\lambda, t^{-1}\tilde{\lambda})$

AS t^{-2h} IF $|h| \leq 1$.

THIS IS TRUE FOR ALL h, AS

FOLLOWS FROM THE FOLLOWING

ALTERNATIVE EXPLANATION:

IF A MASSLESS PARTICLE IS

MOVING IN THE $\vec{n}$ DIRECTION

THEN A ROTATION BY

ANGLE θ AROUND $\vec{n}$ AXIS

IS $(\lambda, \tilde{\lambda}) \to (e^{-i\theta/2}\lambda, e^{+i\theta/2}\tilde{\lambda})$

SO $\lambda, \tilde{\lambda}$ CARRY $-\frac{1}{2}$ OR $+\frac{1}{2}$

OF ANGULAR MOMENTUM

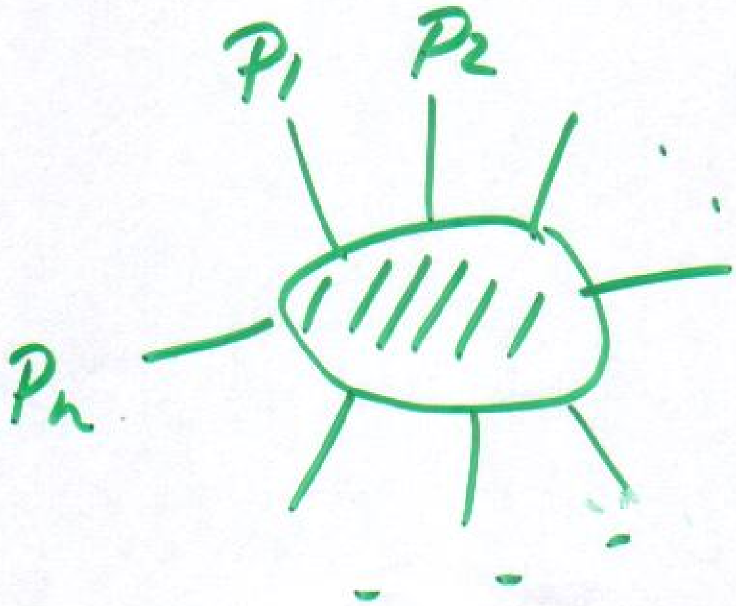
AROUND THE $\vec{n}$ AXIS.

$$\underline{\left(\lambda\frac{\partial}{\partial\lambda} - \tilde{\lambda}\frac{\partial}{\partial\tilde{\lambda}}\right) \psi(\lambda, \tilde{\lambda}) = -2h\,\psi}$$

$$\psi \sim (\tilde{\lambda})^{2h} \qquad \tilde{\lambda}^{2h}(\lambda\tilde{\lambda})^{n}$$

NOW LET US CONSIDER SCATTERING OF MASSLESS PARTICLES IN FOUR DIMENSIONS.

FOR MASSLESS AND SPINLESS PARTICLES, THE SITUATION IS CLEAR:



$$\sum p_i = 0$$

SCATTERING AMPLITUDE IS

A FUNCTION OF THE p_i

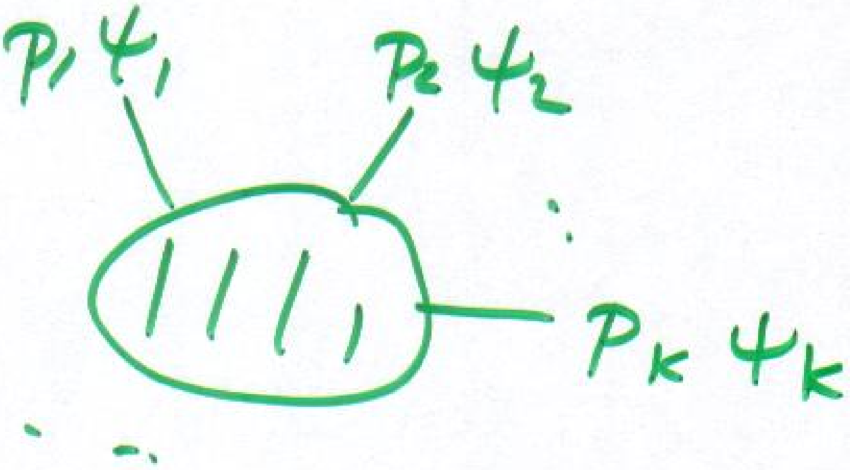
$$A(p_1, \dots, p_n)$$

IN FACT, A FUNCTION OF

THE LORENTZ-INVARIANTS

$$p_i \cdot p_j \, .$$

FOR MASSLESS PARTICLES WITH SPIN, IT IS MORE COMPLICATED:



TEXTBOOKS TELL US TO INTRODUCE FOR EACH PARTICLE OF MOMENTUM p_k A CORRESPONDING WAVE FUNCTION ψ_k AND THEN

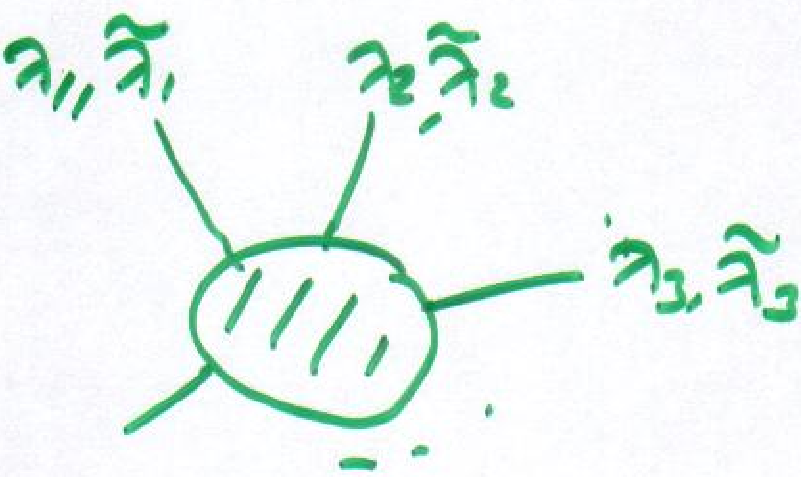
$$A(p_1, \psi_1; p_2, \psi_2; \dots ; p_n, \psi_n)$$

HERE A IS LINEAR IN EACH OF THE ψ_i. TEXTBOOK DESCRIPTIONS OF THE ψ_i ARE DIFFERENT FOR EACH VALUE OF THE SPIN, BUT WE CAN GIVE A MORE UNIFORM DESCRIPTION.

LABEL EACH PARTICLE NOT JUST BY MOMENTUM p_i BUT BY $\lambda_i, \tilde{\lambda}_i$

$$p_{i\,a\dot{a}} = \lambda_{ia} \tilde{\lambda}_{i\dot{a}}$$

$i = 1 \dots n$ $\quad$ n particles



THE WAVEFUNCTIONS ψ_i ARE FUNCTIONS OF $\lambda_i, \tilde{\lambda}_i$ AS ARE THE MOMENTA p_i ...

SO THE SCATTERING AMPLITUDES ARE FUNCTIONS

$$A(\lambda_1, \tilde{\lambda}_1; \lambda_2, \tilde{\lambda}_2; \dots; \lambda_n, \tilde{\lambda}_n)$$

WHICH OBEY, FOR EACH i,

$$\left(\lambda_i \frac{\partial}{\partial \lambda_i} - \tilde{\lambda}_i \frac{\partial}{\partial \tilde{\lambda}_i}\right) A = -2 h_i A$$

h_i = HELICITY OF i^{TH} PARTICLE

$$A\,(p_i, \epsilon_i)$$

AN EXAMPLE OF PHENOMENOLOGICAL IMPORTANCE IS YANG-MILLS THEORY. MULTIJET PRODUCTION AT LHC WILL BE DOMINATED BY TREE LEVEL GLUON SCATTERING AMPLITUDES:

2 JET PRODUCTION

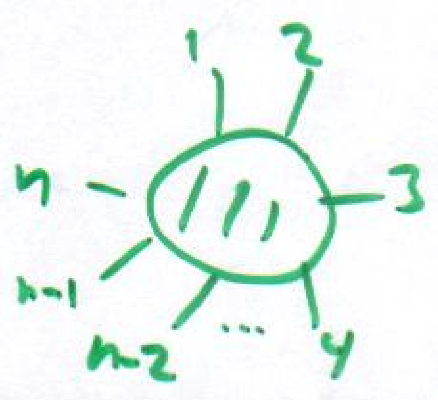
3 JET PRODUCTION

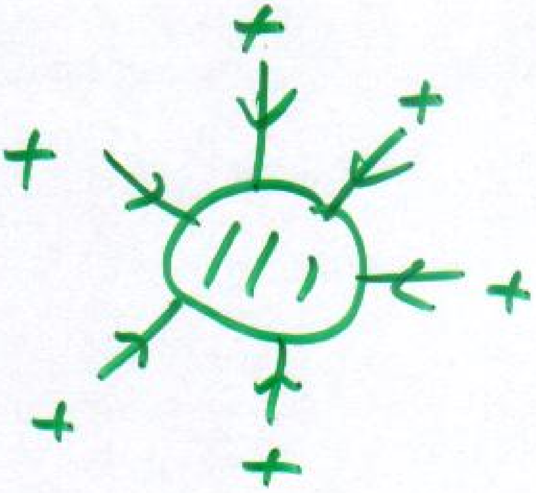
IN THE n GLUON TREE AMPLITUDE, LET'S DISCUSS THE TERM PROPORTIONAL TO

$$\mathrm{Tr}\, T_1 T_2 \cdots T_n$$

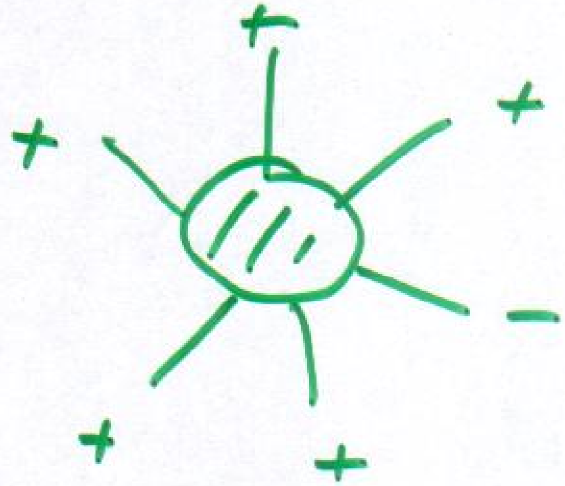
(THE COMPLETE TREE AMPLITUDE IS A SUM OF TERMS PROPORTIONAL TO PERMUTATIONS OF THIS.)

NATURAL CYCLIC ORDER





$= 0$



$= 0$

MHV = MAXIMAL HELICITY VIOLATING

THE FIRST NONVANISHING AMPLITUDE IS THE PARKE-TAYLOR MHV AMPLITUDE... THE ONE WITH JUST TWO GLUONS OF NEGATIVE HELICITY.