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Modes on Rotating Neutron Stars
Successes of Helioseismology

Angular degree \( \ell \)

Rotation profile of the Sun

Frequency

[Thompson et al.]

[Kosovichev et al.]

Solar Sound Speed (SOHO MDI data)
Observed Frequencies in Accreting NSs

- In Persistent Emission
  - kHz QPOs
    - Low-frequency QPOs in "Z" sources
      - 5-7 Hz ("normal branch")
      - 15-50 Hz ("horizontal branch")
  - What else?
    - Very low frequency noise
      - 4U1636-536, 4U 1808-52'
    - ~0.01 Hz QPOs in 4U1608-52'
    - Sidebands [Chakrabarty, Strohmayer and others]
      - Burst oscillations
      - Sidebands [Chakrabarty's talk]

- During Bursts
  - Sco X-1 [van der Klis 1997 review]
  - Low-frequency QPOs in "Z" sources
    - 15-50 Hz ("normal branch")
    - ~0.01 Hz QPOs in 4U1608-52'

### The Zoo of Oscillation Modes

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<th>Frequency</th>
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<td>G-modes</td>
<td>Buoyancy</td>
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<td>R-modes</td>
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<td>~ spin (*)</td>
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<td>imaginary</td>
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Can Any of These Modes Match Observed Frequencies?
G-modes

Surface Waves

Frequency

Displacements mostly transverse

\( \frac{R}{\tau/[(1 + l)]} \sim \eta K \)

\( \frac{\eta R}{(1 + l)l} \)

\( \frac{u}{\tau} \)

\( \frac{\eta}{\tau} \)

Vertical structure

Transverse wavelength

Brunt-Väisälä frequency

Internal waves

Vertical

\( N = \frac{0}{\tau} \)

\( \frac{\eta}{\tau} \)

\( \frac{\eta K}{\tau} \)

\( \frac{\eta}{\tau} \)
0.1\,\text{\text{Ledd}}, \text{mixed H/He, Fe ashes}

\[ \frac{d}{dV} \frac{\eta}{\delta} \approx \zeta N \]

\[ \frac{\eta}{\delta} \sim \zeta N \]

\[ \frac{\eta}{\delta} \sim \zeta N \]

\[ \frac{\text{Brunt depth}}{\delta} = \zeta N \]

\[ \left( \left| \frac{d}{dV} \right| \frac{\eta}{\delta} \right)_{\text{ad}} = \zeta N \]

\text{Buoyancy Sources in NSs}
G-Mode Frequencies

\[ \text{Frequency} = (\text{vertical structure}) \times (\text{transverse wavelength}) \]

- Thermal modes in deep, degenerate ocean \[ \text{Bildsten} \& \text{Cutler '95} \]
- Density discontinuity due to He capture \[ \text{Bildsten} \& \text{Cumming '97} \]
- Thermal modes in upper layers (non-degenerate) \[ \text{C.F. McDermott} \& \text{Taam '87}, f \approx 30-60 \text{ Hz for } l=1 \text{ during burst} \]
- See also \text{McDermott et al.; Strohmayer; Reisenegger \& Goldreich} for other modes

\[ \frac{2}{(I+1)l} \left( \frac{m}{\eta} \right)^{1/2} \] Hz \[ \approx f_{\text{non-degen}} \]

\[ \frac{2}{(I+1)l} \left( \frac{0.1}{X} \right)^{1/2} \] Hz \[ \approx f_{\text{p}} \]

\[ \frac{2}{(I+1)l} \left( \frac{V^{u} \Theta}{16 \sqrt{8L}} \right) \] Hz \[ \approx f_{\text{deep}} \]
G-modes and Rapid Rotation

How rapid is "rapid"?

\( \frac{\gamma}{(1 + 1)} I \)

\[ \begin{align*}
\text{Equations decouple:} & \\
\text{Radial \& Transverse} & \\
\end{align*} \]

Strong buoyancy: \( N^2 \ll \frac{\Omega^2}{\beta} \)

Thin layer: \( \eta \gg \nu \)

"Traditional" approximation

\[ \begin{align*}
\text{Transverse momentum equation:} & \\
\text{[but still slower than breakup spin, \( (\Omega M/R^3)^{1/2} \)]} & \\
\end{align*} \]

\[ \begin{align*}
\text{Coriolis force} & \\
\text{Pressure gradient} & \\
\end{align*} \]

\[ \begin{align*}
2\pi \Delta \times \frac{\partial}{\partial \theta} & = \frac{\partial}{\partial \theta} \left( \frac{1}{\Delta} \frac{\partial}{\partial \theta} \right) \\
\end{align*} \]

\[ \begin{align*}
\text{Konigl, Eichler, \& Cutler, 96} & \\
\text{Bildsten, Pringle, \& Pringle, 78} & \\
\text{Longuet-Higgins, 68} & \\
\end{align*} \]

C-modes and Rapid Rotation
G-modes and Rapid Rotation

Observer sees

Mode frequency

Transverse wavelength

Equatorial confinement

Coriolis force is small when

\[ \frac{2 \pi}{\omega} \gg \frac{R}{\gamma} \left( \frac{\omega}{2 \pi} \right) \]

\[ \frac{R}{\gamma} \sim \frac{\omega}{2 \pi} \]

\[ \frac{2 \pi}{\omega} \approx \cos \theta \]

\[ \omega \approx \frac{m}{50, \mu m} \]

\[ \frac{\omega}{\gamma} \]

\[ \frac{\omega}{2 \pi} \]

\[ \frac{R}{\gamma} \]

\[ \omega \]

\[ \theta \]
G-mode Dispersion Relation

$\lambda \sim q \left( \frac{1}{1+q} \right)$

Eigenvalue $\lambda$

Spin parameter $q = 2\Omega / \omega$

$\lambda \propto \frac{2}{R}$

[Bildsten, GU, & Cutler '96]
G-mode Dispersion Relation

[Bildsten, Cumming, GU, & Cutler '97]

Stellar Spin Frequency (Hz)

Observed Mode Frequency (Hz)

\[ I = 1 \]

\[ Z = 1 \]

\[ H_0 = 30 \text{ Hz} \]

\[ f \]
G-Modes as kHz QPOs?

Bildsten, Cumming, Cutler '97
G-Modes as Low-Frequency QPOs?

\[ \text{Stellar Spin Frequency (Hz)} \]

\[ 10^2 \]

\[ 10^3 \]

\[ 10^4 \]

\[ 10^5 \]

\[ 10^6 \]

\[ 10^7 \]

\[ 10^8 \]

\[ 10^9 \]

\[ 10^{10} \]

\[ 10^{11} \]

\[ 10^{12} \]

\[ \text{Observed Mode Frequency (Hz)} \]

\[ \text{Mode Frequency (Hz)} \]

\[ \text{I}=1 \]

\[ \text{I}=2 \]

\[ \text{m}=-2 \]

\[ \text{m}=-1 \]

\[ \text{m}=0 \]

\[ \text{m}=1 \]

\[ \text{m}=2 \]
What about internal wave (baroclinic) r-modes?

Only true for "surface wave" r-mode

Independent of NS structure

\[
\frac{(I + I)}{2} \text{rotating} = \text{mG}
\]

GWs from NSs mantra:

Restoring force = Coriolis

R-modes
R-mode Dispersion Relation

Slow rotation, "surface" r-modes
Rapid rotation, buoyant r-modes

Spin parameter $\frac{g}{2\Omega^2}$

R-mode Dispersion Relation
Buoyant R-modes

$m=1$

$m=2$
R-mode Dispersion Relation
Modes as Sidebands During Bursts?

30 - 50 Hz sideband in 4U 1728-34 ($f_{\text{spin}}^\text{spin} = 363$ Hz) => need mode with $f \sim 30-50$Hz in rotating frame

Upper sideband?
Modes as Sidebands During Bursts?

30 - 50 Hz sideband in 4U 1728-34 (f_{spin}=363 Hz)
Modes as Sidebands During Bursts?

See much more in Tony Piro's talk.

\[ f_{\text{lower}} \approx 0.2f_0 \]

If upper sideband frequency is

\[ f \approx 0.7f_0 \]

For fundamental frequency \( f_0 \)

\[ f \approx \frac{30}{f_{\text{spin}}} - 50 \text{ Hz in Rotating Frame} \]

\[ \Rightarrow \text{need mode with } \sim 30-50 \text{ Hz in Rotating Frame} \]

30 - 50 Hz sideband in 4U 1728-34 (\( f_{\text{spin}} = 363 \) Hz)
Can the Burst NCO itself Be a Mode?

If burst oscillations are modes, would expect to see:

- Chirp from \( f_{\text{spin}} \) to \( f_{\text{spin}} - 10 \text{ Hz} \) during tail (upper layers)
- Retrograde mode, so observed \( f \) increases in burst's tail
- Mode at \( f_{\text{spin}} - 10 \text{ Hz} \), constant frequency (deep ocean)
- Mode at \( f_{\text{spin}} - 0.5 \text{ Hz} \), constant frequency, smaller amplitude (density discontinuity)

Perhaps a mode at \( f_{\text{spin}} - 10 \text{ Hz} \), constant frequency, smaller amplitude (density discontinuity)

Thermal modes in deep ocean have \( f \approx 0 \text{ Hz} \), but cannot change \( T \) there by 10x to get frequency drift.

Temperature in outer layers changes by 10x, but \( f \approx 50 \text{ Hz} \) there, so frequency drift would be much larger than observed.

\( l \sim 0.1 \), so need a mode with \( f \sim 0 \text{ Hz} \) to \( f \approx \text{few Hz} \).

Amplitude (density discontinuity) smaller.
Summary

Neutron stars potentially have many oscillation modes: if we could identify just one, we'd learn a tremendous amount.

1. Frequency derivatives
2. Ratios of frequencies for fixed radial structure (e.g., \( l=1/l=2 \))
3. Dispersion relations are reasonably well-understood, including when rapidly rotating, and are very constraining.

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