1. Ignition conditions and structure of burning fronts
2. Propagation of burning on a sphere
3. Burning dynamics in magnetic fields
4. Conclusions and connection to observations

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Overview of ignition

- Instability due to high temperature sensitivity of He burning
  \[
  \frac{d \varepsilon_{\text{nuc}}}{dT} > \frac{d \varepsilon_{\text{cool}}}{dT}
  \]

- Type of instability depends on accretion rate:
  \[10^{-8} \quad 10^{-9} \quad 10^{-10} \quad 10^{-11} \quad \text{M}_{\odot}/\text{yr}\]
  Stable -> H/He -> He -> H

- Conditions at ignition (Cumming and Bildsten 00):
  \[T \sim 10^8 \text{ K}, \ y \sim 5 \times 10^8 \ \text{g cm}^{-2}, \ \rho \sim 10^6 \ \text{g cm}^{-3}, \ h \sim 10^2 \text{ cm}, \ t_{\text{nuc}} \sim 0.1 \text{ s}\]
Simultaneous ignition is unlikely: consider local ignition and subsequent deflagration

Rise time $< 1\text{s}$ $\Rightarrow$ front speed $> 3 \times 10^6 \text{ cm/s}$

Various estimates (Fryxell & Woosley 1982)

• **Conduction:** too slow (100 cm/s)

• **Convection:** convective speed $5 \times 10^6 \text{ cm/s}$

  1) Front width = scaleheight; speed $\sim 10^4 \text{ cm/s}$; too slow
  2) Convective diffusion; speed $\sim 10^5 \text{ cm/s}$; too slow
  3) “Wrinkled” front; speed $\leq v_{\text{conv}}$ (upper limit)

• **Consider missing physics:** lift-up and rotation
1. Nonuniform heating generates hydrostatic expansion and hydrodynamic flows (winds)

\[ h_{\text{cold}} \sim 1m \]
\[ h_{\text{hot}} \sim 10m \]

2. Winds transport entropy

3. Winds are confined on the scale set by rotation \( \Delta \) front width

Is rotation important? \( t_{\text{nucl}} \leq t_{\text{rise}} \sim 0.1-1 \text{sec} \gg P_{\text{rotation}} \)

Rotation is not only important to ignition, it is crucial!
Ignition in 2 dimensions

Local ignition is possible only if the temperature perturbation can be sustained on nuclear time scales.

Rotation is able to confine the spread of temperature perturbations.
Rossby radius \( a_R = \sqrt{gH / 2\Omega} \)

\( a_R \) is a typical size of synoptic motions on Earth: \(~1000 \text{ km}, \) on NS \(~1\text{ km} \)

\( a_R/R \approx 0.1 \) for both Earth and NS. Ignition on NS is likely to start on Rossby scale.
Burning front propagation

Front speed: \( \frac{\text{front width}}{t_{\text{nuc}}} = \frac{a_R}{t_{\text{nuc}}} \approx 20 - 60 \text{km/s} \)
• But really, how the entropy was mixed?

To address mixing instability
need 3D: baroclinic, KH.

\[ a_R / t_{nuc} \rightarrow a_R / (t_{nuc} + t_{mix}) \]

• Should faster stars have slower rise times?

Not necessarily -- depends on convective friction

• Does the evolution on

a sphere looks like this? No!
Global model of X-ray bursts

Walls of fire: equator to pole propagation at the end of the rise

Ignition off equator (with latitude-dependence of Coriolis force ignored)

It is possible to ignite the star without creating large asymmetries
**Evolution on a sphere**

- Vortex evolution: two dimensional modeling on a sphere. Spherical shallow-water model. 500 turns of 300 Hz star.

- New in 2D: drift due to variation of Coriolis parameter over the surface, "beta-drift". 
  \[
  \frac{\Delta \Omega}{\Omega} = \frac{g H}{R^2 \Omega^2} = \frac{1}{\varepsilon} = 0.01 \left( \frac{300 \text{Hz}}{v} \right)^2 
  \]

- Rossby wave dispersion 
  \[ v_{phase} = \frac{\beta}{k^2} \]
Ignition on a sphere

- 200 turns of 300 Hz star

- Spreading is preferentially along constant latitude lines. Faster spreading towards equator.
Ignition on a sphere

- 200 turns of 300 Hz star (spherical projection)
Ignition on a sphere

- 300 turns of 300 Hz star, slower burning

- Spreading is preferentially along constant latitude lines. Faster spreading towards equator.
Ignition from equator
Ignition from equator: zonal shear
Ignition from equator: Rossby and Kelvin waves
Long-wavelength mode

\[ v_{\text{phase}} = \frac{\beta}{k^2 + (2n+1)\beta / c_g} \rightarrow \frac{c_g}{2n+1} \]

Frequency of \(~20\) Hz. To match observations need highly baroclinic mode: effective depth \(1/100\) of the scaleheight. How to excite it?

Can the mode exist nonlinearly?
Summary of ignition on a sphere

- Burning can envelop the whole star fast enough for short burst rise times.
- Beta-drift is promising as a frequency signal in the beginning.
- Initial asymmetries are removed extremely well by the spreading: Burning always ignites equator and propagates as “walls of fire” from then on.
- Shallow water simulations allow for growth of equatorially trapped modes. Is such “adiabatic wave ignition” physical? See Tony Piro’s talk. Need real hydro simulations (or primitive equations) to sort this out.
- Long modes are not excited efficiently. In addition, long modes do not live in nonlinear regime. Rossby solitons are the only long-lived coherent structures in the absence of shear flows. But… they move too fast and have large harmonic content.
- Shear flow instabilities for vortex formation in the tail are not ruled out yet.
Magnetic fields -- the final frontier

• Why bother? Magnetic fields are weak!

For some bursters (e.g., SAX 1808) it’s strong enough to make persistent pulses. Also, magnetic field can be wound up, and vortices are ideal for this.

• What B field does to internal vortex evolution?

• What B field does to vortex motion?

• What B field does to modes?

Scalings:

\[ B_{\text{crit}} = \sqrt{4\pi\rho} \quad v_0 = 7 \times 10^{11} G \left( \frac{v_0}{2 \times 10^8 \text{ cm/s}} \right) \left( \frac{\rho}{10^6} \right)^{1/2} \]

Winding up:

\[ \frac{B_x}{B_z} = \frac{x}{H} \quad B_{\text{init}} = 5 \times 10^5 G \left( \frac{v_0}{2 \times 10^8 \text{ cm/s}} \right) \left( \frac{\rho}{10^6} \right)^{1/2} \left( \frac{1 \text{ km}}{a_R} \right) \left( \frac{300 \text{ turns}}{N} \right) \]

Field is important after N turns with initial field

What if B has significant effect?
Magnetic shallow water equations

\[ \frac{\partial}{\partial t} h + \frac{\partial}{\partial x} h v_x = 0 \]

\[ \frac{\partial}{\partial t} v_x = -g \frac{\partial}{\partial x} h + 2\Omega v_y - \frac{B_z}{4\pi\rho} \frac{B_x}{H_0} \]

\[ \frac{\partial}{\partial t} v_y = -2\Omega v_x - \frac{B_z}{4\pi\rho} \frac{B_y}{H_0} \]

\[ \frac{\partial}{\partial t} B_x = v_x \frac{B_z}{H_0} \]

\[ \frac{\partial}{\partial t} B_y = v_y \frac{B_z}{H_0} \]

\[ \alpha = \frac{B_z}{\sqrt{4\pi\rho}} \]

\[ \gamma = \frac{\sqrt{gH_0}}{2\Omega} \]
Magnetic fields -- the final frontier

Forcing of rotating fluid without B field reaction
Forcing of rotating fluid with B field reaction
Magnetic fields -- the final frontier

• What does B field do to internal vortex evolution?

  “Magnetostrophic adjustment”

• New scale: \[ a_B = H \frac{v_g}{v_A} = 5km \left( \frac{10^8 G}{B} \right) \left( \frac{H}{10^2 cm} \right) \]

• What does B field do to vortex motion?

beta-drift oscillations?

• What does B field do to modes?

  Preliminary: Rossby, gravity waves almost unchanged for weak fields (modes don’t wind up the field). Westward “magnetostrophic” wave becomes Rossby wave.
CONCLUSIONS AND FUTURE WORK

1. Burning front propagation based on geostrophic circulations is being tested by full numerical simulations, so far successfully. MultiD simulations also allow to find the baroclinic structure of modes in order to construct equivalent shallow-water simulation.

2. Burning on sphere either starts on the equator or propagates to the equator and then in the form of “walls-of-fire” to the poles. Initial asymmetries are erased in all cases.

3. In future attempt to simulate nonuniform burning material distribution on the star.

4. Investigate dynamical effects of B-fields -- maybe important both to magnetic accretors and “nonmagnetic” bursters in order to bring the shell to corotation with star. Finally, time to get a signal from simulations!