The Role of Magnetic Reconnection in Angular Momentum Transport

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**Disk MRI:**

MRI mechanism: angular momentum transfer (AMT) by magnetic fields **within the disk**.

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Angular momentum transfer by coronal magnetic loops.

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Coronal vs. Disk MRI

Q: Which mechanism (disk MRI or coronal MRI) is the Dominant Angular Momentum Transport Process (DAMTP)?

- Magnetic stress in the disk:
  \[ G_{\text{disk}} \sim 2\pi r < B_r B_\phi >_{\text{disk}} \quad H \sim B_{\text{disk}}^2 H \]

- Magnetic stress in the corona:
  \[ G_{\text{cor}} \sim 2\pi r < B_r B_\phi >_{\text{cor}} \quad H_B \sim B_{\text{cor}}^2 H_B \]

- Thus,
  \[ \frac{G_{\text{cor}}}{G_{\text{disk}}} \sim \frac{B_{\text{cor}}^2}{B_{\text{disk}}^2} \frac{H_B}{H} \]

- Two AMT ingredients:
  (1) Characteristic magnetic field strength \((B_{\text{cor}} \text{ or } B_{\text{disk}})\);
  (2) Magnetic scale height \((H_B \text{ or } H)\).

Coronal MRI dominates if \(H_B \gg H\), while \(B_{\text{cor}} \sim B_{\text{disk}}\).

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Coronal Magnetic Field $B_{\text{cor}}$: 
Flux Emergence from MRI-active disk

• Typical coronal magnetic field strength, $B_{\text{cor}}$, and hence the overall coronal power and torque depend on the normalization of $F(L, \theta)$.

• This, in turn, depends on the rate and form of buoyant flux emergence from the disk.

• 3D MHD simulations of MRI-turbulent stratified disks
  (e.g., Brandenburg et al. 1995; Stone et al. 1996; Miller & Stone 2000; Hirose et al. 2007; Blaes et al. 2007)

• **Key Question:**
  What controls flux emergence in a stratified MRI-turbulent disk?

• **Suggestion:**
  Competition between Parker and other secondary instabilities.

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  – parasitic instabilities (e.g., KH) of *Goodman & Xu (1994)*, ⇒ 3D MHD turbulent cascade;

(parasitic (Goodman & Xu 1994))
Nonlinear Fate of MRI Channel Flows

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  - resistivity ⇒ tearing mode;

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  – resistivity ⇒ tearing mode;
  – gravitational stratification ⇒ Parker instability ⇒ flux emergence ⇒ magnetized corona.
What is the characteristic magnetic scale-height $H_B$?

*More generally:*

- what is the distribution of magnetic energy density $\bar{B}^2(z)/8\pi$ with height $z$ above the disk? (for $H \ll z \ll R$)
- If there is a characteristic magnetic scale $H_B$, how large is it compared with $H$ and $R$? What determines it?
- Or, if $\bar{B}^2(z)/8\pi$ is a power law, what is the power-law exponent?
- What is the distribution of magnetic dissipation with height?

*Even more generally:*

- What is the distribution $F$ of coronal magnetic loops in sizes, field strengths, etc.? Is there inverse cascade of coronal loops?

- How does $\bar{B}^2(z)/8\pi$ control coronal AMT?
- What is the role of magnetic reconnection?

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• **Goal:** build statistical description of force-free coronal magnetic field above turbulent accretion disk.

• **Program** (see also Tout & Pringle 1996):
  – Represent corona by ensemble of elementary magnetic loops characterized by footpoint separations: $\Delta r, \Delta y = r \Delta \phi$.
  – Introduce *loop distribution function* $F(\Delta r, \Delta y)$.
  – Derive equations of motion for loops in this parameter space.
  – Derive the *Loop Kinetic Equation* for $F$.
  – Obtain a *Statistical Steady State*.

• **Scales of Interest:**
  – temporal: $\Omega^{-1} \ll \tau \ll T_{\text{accr}}$
  – spatial: $H \ll l \ll R$

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Processes governing individual loop evolution:

- **stressing:**
  (increasing magnetic energy and magnetic scale-height $H_B$)
  - emergence (and submergence) of loops into corona
  - stretching by Keplerian shear
  - random footpoint walk due to disk turbulence

- **relaxation:**
  (dissipating magnetic energy and bringing the field closer to potential)
  - reconnection between loops (manifested as flares)
    $\Rightarrow$ **inverse cascade:** formation of larger structures.
    *(Tout & Pringle 1996)*

Overall, a magnetically-active corona can be described as

**A BOILING MAGNETIC FOAM!**

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THE LOOP KINETIC EQUATION

• Final Result: Loop Kinetic Equation for the distribution function $F$:

$$\frac{\partial F}{\partial t}(A, t) = S(A) + (D_r \frac{\partial^2}{\partial \Delta r^2} + D_y \frac{\partial^2}{\partial \Delta y^2}) F(A) - \frac{\partial}{\partial \Delta y}(F \dot{\Delta} y) + \dot{F}_{\text{rec}}(A)$$

• (1) Flux Emergence $\Rightarrow$ source term $S(A)$

• (2) Random footpoint motions $\Rightarrow$ diffusion term

• (3) Keplerian shear $\Rightarrow$ advection term $- \frac{\partial}{\partial \Delta y}(F \dot{\Delta} y)$, with $\dot{\Delta} y = -3/2 \Omega \Delta r$.

• (4) Reconnection between loops $\Rightarrow \dot{F}_{\text{rec}}$

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Reconnection acts as a binary collisions between particles in a gas:

- Footpoint positions are preserved during interaction ⇒ reconnection rules (similar to conservation laws).

- Reconnection is represented by a non-linear integral operator \( \sim \) collision integral in Boltzmann’s eqn: (c.f., Tout & Pringle 1996)

\[
\dot{F}_{\text{rec}}(A) = \dot{F}_{\text{coll},-}(A) + \dot{F}_{\text{coll},+}(A)
\]

\[
\dot{F}_{\text{coll},-}(A) = - \int dB \ Q_{AB} \ F(A) \ F(B)
\]

\[
\dot{F}_{\text{coll},+}(A) = \frac{1}{2} \int \int dC \ dD \ Q_{CD \rightarrow A} \ F(C) \ F(D)
\]

\( Q_{AB} = q \Omega \sigma_{AB} \) is the reconnection-event rate.

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Results: Statistical Steady State

Projected loop length: \( L = \sqrt{\Delta r^2 + \Delta y^2} \)

Orientation angle with respect to \( \phi \)-direction: \( \theta \)

\( q = \infty \) – no Keplerian shear; \( q \ll 1 \) – strong shear

(semi-circular loops)

\[
F(L, \theta) \sim L^{-\alpha(\theta)}
\]

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• Energy associated with a single loop:

\[ \mathcal{E}(L) = 2E_{\text{magn}}(L) = \frac{\Delta \Psi}{4\pi} \int_0^L B_x(z = 0; L') \, dL' \]

\[ = \frac{\Delta \Psi}{4\pi} \int_0^L B_{\text{top}}(L') \, dL' \]

\[ = \frac{\Delta \Psi^2}{4} \int_0^L \int_0^{L''} F(L'') \, dL'' \, dL' . \]

• Total magnetic energy in the corona:

\[ E_{\text{tot}} = \frac{1}{2} \int_0^\infty \bar{F}(L) \mathcal{E}(L) \, dL \]

\[ = \frac{\Delta \Psi^2}{4} \int_0^\infty dL \int_0^L dL' \int_0^{L'} \bar{F}(L) \bar{F}(L') . \]

• Total magnetic torque per unit disk area:

\[ G = -\Delta \Psi B_0 \frac{1}{8\pi} \int \int dL \, d\theta \, F(L, \theta) b_{\text{top}}(L) \, L \sin 2\theta . \]

• Energetics of individual reconnection events:

\[ E_{\text{flare}}^{A+B \rightarrow C+D} = E(A) + E(B) - E(C) - E(D) \]

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Results: Magnetic Energy and Torque

Coronal Angular Momentum Transport:

![Graph showing Coronal Angular Momentum Transport]

Total Coronal Magnetic Energy:

![Graph showing Total Coronal Magnetic Energy]

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Reconnection is too efficient: magnetic field is close to potential, $E_{\text{magn}} \simeq E_{\text{pot}}$; free magnetic energy and magnetic dissipation are small.

Reconnection is forbidden: forest of open field lines, $E_{\text{magn}} \simeq E_{\text{open}} \gg E_{\text{pot}}$; free magnetic energy is large but dissipation is not allowed.

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SUMMARY

• Magnetic field in Accretion Disk Corona (ADC) contributes to Angular Momentum Transfer (Coronal MRI), especially if $H_B \gg H$.

• Magnetic flux emergence driven by magnetic buoyancy/Parker instability in a stratified disk leads to the formation of ADC.

• Rate of flux emergence is set by competition between Parker instability and other parasitic instabilities feed by the primary nonlinear MRI mode.

• ADC as a boiling magnetic foam.

• Statistical description of ADC magnetic field: ensemble of loops, Loop Kinetic Equation ...

• Magnetic reconnection controls $\bar{B}^2(\bar{z})/8\pi$, coronal magnetic energy and torque.

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