Self Sustaining Cycles in Zero net flux MRI Turbulence

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Why should we care about zero mean field SBs?

The “classical” MRI is found with a mean vertical field across the box $B = B_0 e_z$

- Origin of the imposed magnetic field?
- Channel flow solution? (BCs effect? see Bodo et al. 2008)

*Channel flow “bursts”*

$\langle B^2 \rangle$ time history with a mean vertical field

Simulation with no net flux?
Turbulence in zero mean field boxes: Open questions

No Linear theory for the zero net flux case

- How turbulence is maintained?
- Dynamo action? (Magnetic field generated by turbulent motions)
- How do we explain the Pm dependancy?
- Large scale or small (dissipation) scale mechanism?

“MRI type” instability

“Dynamo”

Fromang et al. (2007)
The quest for a non linear mechanism

Non linear steady solution (Rincon et al. 2007) in Couette flows (no slip, perfectly conducting walls in the y=r direction)

Non axisymmetric solution

Involves a large scale $B_\phi(z)$

But no steady solution of this kind in the shearing box!
\[ \partial_t \mathbf{u} + S y \partial_x \mathbf{u} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla \psi - 2\Omega u_x \mathbf{e}_y + (2\Omega - S)u_y \mathbf{e}_x + \mathbf{B} \cdot \nabla \mathbf{B} + \nu \Delta \mathbf{u} \]

\[ \partial_t \mathbf{B} + S y \partial_x \mathbf{B} = SB_y \mathbf{e}_x + \nabla \times (\mathbf{u} \times \mathbf{B}) + \eta \Delta \mathbf{B} \]

\[ \nabla \cdot \mathbf{u} = 0 \]

\[ \nabla \cdot \mathbf{B} = 0 \]

**Incompressible approximation (no sound/density waves)**

**Physical viscosity and ohmic resistivity:**

- Reynolds number \( Re = \frac{SH^2}{\nu} \)
- Magnetic Prandtl \( Pm = \frac{\nu}{\eta} \)

**Numerics:**

- 3D Spectral (Fourier) method with remap (e.g. Umurhan & Regev 2004)
- MPI parallelization (FFTW 2.1.5)
An azimuthal field cycle in shearing boxes?

Shearing boxes simulations seem to show a strong azimuthal field with a vertical dependancy.

Fourier Analysis of $B_\phi(k_0 = \frac{2\pi}{L_z}e_z)$ shows regeneration cycles with $T \sim 50$ S$^{-1}$.

No cycle in the transport coefficients.
Behaviour during one cycle

Simulation in a “tall” box ($L_z=2L_r$)
Re=1600, Pm=4 for 50 S$^{-1}$

3D Plot of $B_\phi$
Studying one cycle...

Budget for a pure $\exp(ik_0z)$ mode:

\[
\frac{\partial \hat{B}_\phi}{\partial t} = S \hat{B}_r - ik_0 \hat{E}_r - \eta k_0^2 \hat{B}_\phi
\]

\[
\frac{\partial \hat{B}_r}{\partial t} = ik_0 \hat{E}_\phi - \eta k_0^2 \hat{B}_r
\]

with \( \hat{E} = (u \times B) k_0 \)

Involves coupling between various other modes

One $B_\phi$ cycle from a numerical simulation

$E_r$ acts always as a resistive term

$E_\phi$ is responsible for the cyclic behaviour
Non axisymmetric origin of the emfs

Which waves contribute to the EMFs?

Contribution of non axisymmetric wave numbers \( n_\phi \) to \( E_\phi \)

\( E_\phi \) comes from the coupling of the largest non axisymmetric modes

Same conclusion for \( E_r \) (not shown)
Shearing waves linear response (I)

Let’s consider the linear and non axisymmetric response to a large scale field

\[ B = B_0 \cos(k_0 z)e_\phi \]

The “largest” linear shearing waves are written:

\[ u = \bar{u}(z) \exp[i(k_\phi x + k_r(t)y)] \quad \text{with} \quad k_\phi = \frac{2\pi}{L_\phi} \]
\[ b = \bar{b}(z) \exp[i(k_\phi x + k_r(t)y)] \quad \text{with} \quad k_r = -Stk_\phi \]

The axisymmetric EMF profile associated to one shearing waves is:

\[ E(z) = \frac{1}{2} \Re[\bar{u}(z) \times \bar{b}^*(z)] \]

To quantify the “quasi linear” feedback, we compute the correlations

\[ \Gamma_\phi = \int B_\phi(-\partial_z E_r) \, dz \]

\[ \Gamma_r = \int B_\phi \partial_z E_\phi \, dz \]

Having in mind the large scale field equations:

\[ \partial_t B_\phi = SB_r - \partial_z E_r \]
\[ \partial_t B_r = \partial_z E_\phi \]
Shearing waves linear response (II)

Result from a linear computation with a random set of shearing waves

\[ \mathbf{B} = B_0 \cos(k_0 z)\mathbf{e}_\phi \]

Shearing waves always have a resistive effect on \( B_\phi \)

Shearing waves linear response (II) and is reversed for larger field strengths. This is consistent with the cycles observed in the previous section (see text).

Shearing waves always have a resistive effect on \( B_\phi \)

Shearing waves linear response (II) -- The effect on \( B_r \) get reversed for strong enough \( B_\phi \)
We assume a “turbulent resistivity” closure model for the emfs, following the linear properties of the shearing waves:

\[
\begin{align*}
\partial_t B_\phi(t) &= S B_r(t) - \beta B_\phi(t - t_L) \\
\partial_t B_r(t) &= \gamma B_\phi(t - t_L) \left[ 1 - \frac{|B_\phi(t - t_L)|}{B_{\text{Rev}}} \right]
\end{align*}
\]

\[\text{Lag time } t_L \approx S^{-1} \]
\[B_{\text{Rev}} \approx 0.1\]

Cycles are reproduced with \(T \approx 50 \, S^{-1}\)
Requires \(\gamma \sim \beta \sim \alpha_{SS}\)
Summary: MRI-Dynamo cycle

- Axisymmetric $B_\phi$
- Axisymmetric $B_r$
- Mean Shear
- "MRI Type" amplification
- $E_r = "turbulent resistivity"
- $E_\Phi = "Dynamo"
- Shearing waves
- Non Linear coupling
- Non axisymmetric Seed
- Turbulent Cascade
- Non Linear feedback?
Conclusions

- The MRI naturally provides a (non linear) dynamo feedback when a non homogeneous $B\Phi$ is imposed.
- Explains the existence of a large scale cycle, with no dependance on the dissipation scales (should work even with very large Re, Rm).
- Dynamo feedback due to an anisotropic resistivity (not an $\alpha$ effect).

However

- No precise understanding of the origin of the non axisymmetric seed.
- How the Prandtl number enters this problem?
- Is this mechanism also responsible for the turbulent transport of angular momentum?
Studying one cycle (phases)...

**Budget for a pure exp(\(ik_0z\)) mode:**

\[
\frac{\partial \hat{B}_\phi}{\partial t} = S \hat{B}_r - ik_0 \hat{E}_r - \eta k_0^2 \hat{B}_\phi \\
\frac{\partial \hat{B}_r}{\partial t} = ik_0 \hat{E}_\phi - \eta k_0^2 \hat{B}_r
\]

with \(\hat{E} = (u \times \hat{B})k_0\)

**Involves coupling between various other modes**

One \(B_\phi\) cycle from a numerical simulation

**Er acts always as a resistive term**

**E_\phi is responsible for the cyclic behaviour**