Burst Oscillations in SAX J1808.4-3658
Alfven-Rossby Waves in the Neutron Star Ocean?

Frederick K. Lamb and Yun Chen
University of Illinois at Urbana-Champaign
Properties of X-ray Oscillations in SAX J1808.4-3658

Oscillation during accretion-powered emission —

- r.m.s. amplitude ~ 3%–5%
- frequency = 401 Hz  =>  spin rate = 401 Hz

Oscillation during burst rise —

- r.m.s. amplitude ~ 3%–5%
- increases from ~ 396 Hz to ~ 403 Hz in ~ 0.2 seconds
- 10 times faster than in any other burst source

No detectable oscillations during burst peak

Oscillation during burst decay —

- r.m.s. amplitude ~ 3%–5%
- $\nu_{\text{burst}} = \nu_{\text{osc}} + 6 \text{ mHz}$
Rossby, Alfvén, and Alfvén-Rossby Waves in Neutron Star Oceans

\( \Omega \neq 0, \ B = 0 \Rightarrow \) Rossby waves. Restoring forces are

- Coriolis force
- Pressure-gradient force

\( \Omega = 0, \ B \neq 0 \Rightarrow \) Alfvén waves. Restoring forces are

- Magnetic tension force
- Pressure-gradient forces

For \( \Omega \neq 0, \ B \neq 0 \Rightarrow \) Alfvén-Rossby waves. Forces are

- Coriolis force
- Magnetic tension force
- Pressure-gradient forces
Alfvén-Rossby Waves in the Oceans of Magnetic Neutron Stars

Assumptions and approximations —

• The crust (on which the ocean rests) is spherical
• Ocean depth $h \sim 10 \text{ m} \ll R \sim 10^6 \text{ m}$
• The ocean is a uniform, incompressible fluid with no resistivity or viscosity (ideal MHD approximation)
• $\vec{B} = \text{force-free in ocean, } B_r = 0 \text{ at top and bottom}$
• $M \approx 1.5M_\odot, R \approx 10^6 \text{ cm, } \rho \approx 10^6 \text{ g cm}^{-3}$

Waves in the ocean are —

• Rossby-like, with frequencies related to $\Omega$
• Alfvén-like, with frequencies related to $R/V_A$
Ideal MHD Equations for the Ocean

Describe the velocity and magnetic fields by the stream functions $\Psi$ and $\chi$

$$v^\theta = -\frac{1}{r \sin \theta} \frac{\partial \Psi}{\partial \phi} \quad v^\phi = \frac{1}{r} \frac{\partial \Psi}{\partial \theta} \quad B^\theta = -\frac{1}{r \sin \theta} \frac{\partial \chi}{\partial \phi} \quad B^\phi = \frac{1}{r} \frac{\partial \chi}{\partial \theta}$$

Define the “Poison Bracket” $\{ , \}$ by

$$\{a, b\} = \frac{1}{r^2 \sin \theta} \left[ \frac{\partial a}{\partial \theta} \frac{\partial b}{\partial \phi} - \frac{\partial a}{\partial \phi} \frac{\partial b}{\partial \theta} \right]$$

Then the MHD equations become

$$\frac{\partial}{\partial t} \nabla^2 \Psi + 2\Omega \frac{\partial}{\partial \phi} \Psi = \{\nabla^2 \Psi, \Psi\} - \frac{1}{4\pi \rho} \{\nabla^2 \chi, \chi\}$$

$$\frac{\partial}{\partial t} \chi = \{\chi, \Psi\}$$

$$\frac{\partial \rho}{\partial t} = \{\rho, \Psi\}$$
Solving the MHD Equations for Alfvén-Rossby Waves in Neutron Star Oceans

- Using spectral methods, we have computed the velocity fields, magnetic fields, and frequencies of the linear Alfvén, Rossby, and hybrid Alfvén-Rossby waves in the oceans of magnetic neutron stars.

- The linear waves are specified uniquely by their velocity patterns and canonical momenta.

- We have computed their quasi-linear hydrodynamic and magnetohydrodynamic evolution.

- We have also found families of stable fully nonlinear Alfvén-Rossby waves for any amplitude.
Excitation of Alfvén-Rossby Waves

- Large-scale disturbances of the ocean, such as may occur during an X-ray burst, preferentially launch Alfvén-like waves, which all propagate in the prograde direction as seen in the corotating frame.

- The (2,1) wave is typically excited to a much larger amplitude than the other Alfvén-like waves.

- For spin rates ~ 400 Hz and magnetic fields ~ $10^8$ G, the magnetic and velocity disturbance of the (2,1) wave rotates with a frequency ~ 10 mHz higher than the star’s spin frequency.
Example of an Alfvén-Rossby Wave

The velocity disturbance $\vec{v}$ given by $\Psi \propto$ a mixture of $Y_{2,0}, Y_{2,\pm 1}, Y_{2,\pm 2}$, in the background magnetic field $\vec{B}_0$ given by $\chi \propto Y_{1,1}$, tilted 45 degrees from the rotation axis.
Frequency vs. Magnetic Field Strength for Rossby-Type and Alfvén-Type Modes

(2,1) Rossby-type mode

(2,1) Alfvén-type mode
Evolution of Rossby- and Alfvén-Type Hybrid Wave Amplitudes
Conclusions

• Large-scale disturbances of the ocean typically excite Alfvén-type waves to amplitudes 10 to 100 times larger than they excite Rossby-type waves

• The amplitude in the (2,1)Alfvén-type wave is typically 3 to 10 times larger than the amplitudes of the other Alfvén-type waves excited

• For the conditions expected in the ocean of SAX J1808.4–3658, the (2,1)Alfvén-type wave has frequency ~ 10 mHz higher than the spin frequency
Conclusions (cont’d)

- We therefore suggest that perturbations caused by the (2,1) Alfvén-type wave may be responsible for the weak oscillations with frequencies ~ 10 mHz higher than the star’s spin frequency observed in the tails of X-ray bursts from SAX J1808.4–3658.

- Waves like those we have studied may also be involved in producing the oscillations observed in the tails of X-ray bursts from other neutron stars.