

Electroweak symmetry breaking from dimensional deconstruction

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Abstract

We propose a new class of four-dimensional theories for natural electroweak symmetry breaking, relying neither on supersymmetry nor on strong dynamics at the TeV scale. The new TeV physics is perturbative, and radiative corrections to the Higgs mass are finite. The softening of this mass occurs because the Higgs is an extended object in theory space, resulting in an accidental symmetry. A novel Higgs potential emerges naturally, requiring a second light $SU(2)$ doublet scalar. © 2001 Published by Elsevier Science B.V.

1. Introduction

Experiments beginning later this decade will probe the fundamental mystery of the weak interactions, the origin of electroweak symmetry breaking. The standard model, a theory with a fundamental scalar field that implements the Higgs mechanism, is almost certainly incomplete — quadratically divergent radiative corrections to the Higgs mass suggest that new physics is required at TeV energies to stabilize the weak scale. To date theories for this stabilization can be grouped into two categories: those which rely on new strong dynamics or compositeness near a TeV (such as technicolor, composite Higgs, or theories with a low fundamental Planck scale), and those which are perturbative, using low-scale supersymmetry. In this

Letter we propose a third category: a class of non-supersymmetric theories with a light Higgs, in which physics is entirely perturbative at the TeV scale, yet the Higgs mass radiative corrections are finite.

Aside from supersymmetry there have been few ideas for ensuring a light boson on purely symmetry grounds. Two directions are spontaneously broken accidental global symmetries, [1] that can produce pseudo-Nambu–Goldstone bosons, and gauge symmetries that protect vector boson masses. Neither of these ideas seem directly relevant: the Higgs doesn't look much like a Nambu–Goldstone boson (with non-derivative quartic, gauge, and Yukawa couplings).³ Neither does it look like a boson with spin. Consequently neither of these avenues has produced a viable theory without the need for fine-tuning. Using the idea of dimensional deconstruction [3], we will see that these two ideas are in fact related to each other in a way which allows us to construct realistic theories.

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³ It is possible to make a pseudo-Nambu–Goldstone boson look like a Higgs by fine tuning its mass term. This is the idea behind composite Higgs models [2].

To motivate the connection, we briefly detour into five dimensions.

Consider an $SU(k)$ gauge theory in five dimensions compactified on a circle of radius R . At distances large compared to R , the theory appears four-dimensional, with an $SU(k)$ gauge symmetry. The zero mode of the five-dimensional gauge potential decomposes into the four-dimensional $SU(k)$ gauge bosons and a real scalar field ϕ in the adjoint representation. ϕ is associated with the non-trivial Wilson loop $W = P \exp(i \int dx_5 A_5)$ around the fifth dimension. Classically, ϕ is massless. At 1-loop, ϕ picks up a quadratically divergent mass in the low-energy four-dimensional effective theory. However, at energies much larger than $1/R$, ϕ is really A_5 , the fifth component of the gauge field. So, by the higher-dimensional gauge invariance, there can't be any contribution to the mass of ϕ from energies much higher than $1/R$, and therefore the quadratic divergence of the low-energy theory must be cut off at the scale $1/R$. How is any mass for ϕ generated? Since the Wilson line is gauge covariant, gauge invariance does not forbid the operator $|\text{tr } W|^2$, that contains a mass for the zero mode of A_5 in its expansion. Since this is a non-local operator, five-dimensional locality guarantees that it can not be generated with a UV divergent coefficient. This is interesting, because the theory is non-supersymmetric and even perturbative at the scale $1/R$, and yet the correction to the ϕ scalar mass is completely finite.

In the Section 2 we “deconstruct” this seemingly higher-dimensional mechanism. As we will see, the extra dimension is not at all essential, and the physics can be understood in purely four-dimensional terms. The light scalar is a pseudo-Nambu–Goldstone boson, an object very familiar to gauge theory model builders. Nevertheless, the higher-dimensional picture will be a very useful guide as we explore how this idea can be used as a starting point to provide a new way to stabilize the Higgs mass in the standard model. There are many apparent obstacles to doing this. If the Higgs is to be associated with components of a higher-dimensional gauge field, how can we get it out of the adjoint representation? How can we get a negative mass squared and a reasonably large quartic coupling, so the Higgs gets a vev and the physical Higgs particle is sufficiently heavy?

We will see that these issues are very naturally resolved in a six-dimensional theory. Non-adjoint

scalars can be generated by enlarging the gauge group in the six-dimensional bulk. Having special sites where the gauge group is not enlarged allows us to give negative mass squared to the Higgs. Finally, and perhaps most interesting, the six-dimensional gauge kinetic energy contains non-derivative interactions which become a quartic coupling term between our four-dimensional Higgs and another scalar doublet, stabilizing the potential.

If our theory were truly six-dimensional, we would have the usual higher-dimensional problems to contend with. Why is the radius stabilized near the TeV scale? What happens near the cutoff of the higher-dimensional gauge theory? But in deconstruction, extra dimensions are used purely as inspiration, and may be discarded at the end, together with all the additional restrictions they imply. This allows us to build realistic theories of electroweak symmetry breaking in four dimensions with no higher-dimensional interpretation whatsoever. The new feature of these theories is that the physics of electroweak symmetry breaking remains perturbative and insensitive to high-energy details up to a cut-off scale much larger than a TeV without the need for any fine-tuning.

2. Deconstruction

We wish to deconstruct the toy 5D theory we have just described along the lines of [3]. Since the radiative stability of the Higgs is a low energy problem, we may restrict our discussion to the low energy effective Lagrangian of [3], described by a “condensed moose” diagram. This corresponds to putting the fifth dimension on a lattice with N sites $i = 1, \dots, N$ with periodic identification of site i with site $i + N$ [3–5]. On each site there is an $SU(k)$ gauge group, and on the link pointing from the i th to the $(i + 1)$ -th site, there is a non-linear sigma model field $U_i = \exp(i\pi^a T_a / f)$. Under the $SU(k)^N$ gauge symmetry the link fields transform as $U_i \rightarrow \mathbf{g}_i U_i \mathbf{g}_{i+1}^{-1}$. The effective Lagrangian is

$$\mathcal{L} = -\frac{1}{2g^2} \sum_{i=1}^N \text{tr } F_i^2 + f^2 \sum_{i=1}^N \text{tr}[(D_\mu U_i)^\dagger D^\mu U_i] + \dots, \quad (2.1)$$

where the covariant derivative is $D_\mu U_i \equiv \partial_\mu U_i - iA_\mu^i U_i + iU_i A_\mu^{i+1}$ and the dots represent higher-dimension operators that are irrelevant at low energies.⁴ If the gauge couplings are turned off, there is no coupling between the U 's at different sites and the theory has a large $SU(k)^{2N}$ accidental “chiral” global symmetry

$$U_i \rightarrow L_i U_i R_{i+1}^\dagger, \quad (2.2)$$

where L_i, R_i are independent $SU(k)$ matrices. This is spontaneously broken down to $SU(k)^N$, resulting in N adjoint Nambu–Goldstone boson multiplets.

The gauge couplings preserve only the $SU(k)^N$ subgroup of the global symmetry (2.2) where $L_i = R_i$. Using the gauge freedom we can almost go to a unitary gauge where all the U_i are set to one. The $SU(k)^N$ gauge theory is higgsed to the diagonal $SU(k)$ eating $N - 1$ of the adjoint Nambu–Goldstone bosons along the way. The remaining Nambu–Goldstone boson is associated with the product $U_1 U_2 \cdots U_N$ which transforms homogeneously under \mathfrak{g}_1 gauge transformations and can not be gauged to unity. This operator is the discretization of the Wilson line in the continuum case. The linear combination $\phi = (\pi_1 + \cdots + \pi_N)/\sqrt{N}$ of Nambu–Goldstone bosons is classically massless, and transforms as the adjoint under the surviving diagonal $SU(k)$ gauge group.⁵ This field corresponds to the zero mode of A_5 . Because (2.2) is broken by the gauge interactions, ϕ gets a mass from loop effects.

We can characterize the breaking of (2.2) through the introduction of “spurions” q_i . We assign to q_i a transformation law so that the covariant derivative transforms homogeneously. Taking

$$D_\mu U_i = \partial_\mu U_i + iA_{\mu i} U_i - U_i q_i A_{\mu(i+1)} q_i^\dagger, \quad (2.3)$$

we can read off the transformation law

$$U_i \rightarrow L_i U_i R_{i+1}^\dagger, \quad (2.4)$$

$$A_i \rightarrow L_i A_i L_i^\dagger, \quad (2.5)$$

$$q_i \rightarrow R_{i+1} q_i L_{i+1}^\dagger. \quad (2.6)$$

⁴ Similar models have been considered in other contexts, including models of CP violation [6] and quantum field theory final examinations [7].

⁵ These objects were also noticed by [5].

There is also a separate symmetry $U(1)^N$ symmetry under which

$$q_i \rightarrow e^{i\alpha_i} q_i. \quad (2.7)$$

Note that under a gauge transformation $U_i q_i \rightarrow L_i U_i q_i L_{i+1}^\dagger$. After using q_i to determine the symmetry properties, we set $q_i = 1$.

Now we can discuss the radiative corrections to the ϕ mass. The spurious symmetries tightly constrain the sorts of operators that can be generated. The leading non-trivial operator involving only the U 's is

$$\mathcal{O} = f^4 |\text{tr}(U_1 q_1 \cdots U_N q_N)|^2. \quad (2.8)$$

The expansion of this operator gives a mass for ϕ . What do we expect for the coefficient of this operator from matching to the UV theory? The spurious symmetries ensure that the coefficient is at least as small as $(g^2/16\pi^2)^N$ which is negligible compared to the low energy contribution from radiative corrections.

Recall that for the purposes of power-counting [8, 9], the U_i have scaling dimension 1, and so \mathcal{O} has scaling dimension $2N$. Therefore, for $N > 2$, there is no divergent counterterm required for this operator — all dependence on the cut-off may be absorbed in renormalizations of other parameters in the theory (like f). For the special cases $N = 1, 2$ a quadratic and logarithmic divergence appears, respectively. The ϕ mass is already finite for $N = 3$ sites, where the model bears no resemblance to an extra dimension.

Armed with this power-counting argument, let us calculate the leading 1-loop order low energy contribution to the ϕ potential. The analysis is most efficiently done with the Coleman–Weinberg formalism. Turning on a background value for ϕ corresponds to taking

$$U_i = e^{i\phi/(f\sqrt{N})}. \quad (2.9)$$

The \sqrt{N} has been inserted to make ϕ a canonically normalized field.

For simplicity, we consider the case of an $SU(k = 2)$ theory. Then we can always choose ϕ to point in the σ^3 direction, $\phi = |\phi|\sigma^3$. The Coleman–Weinberg potential from gauge boson loops is

$$V(\phi) = \frac{3\Lambda^2}{32\pi^2} \text{tr} M^2(\phi) + \frac{3}{64\pi^2} \text{tr}(M^2(\phi))^2 \log \frac{M^2(\phi)}{\Lambda^2}, \quad (2.10)$$

where $M^2(\phi)$ is the mass matrix for the N gauge boson multiplets in the presence of the background (2.9) and Λ is the UV cut-off. The discrete translation symmetry of the theory allows diagonalization of the mass matrix by discrete Fourier transform. The resulting eigenvalues for the charged gauge bosons are:

$$m_n^2(\phi) = 4g^2 f^2 \sin^2 \left(\frac{n\pi}{N} + \frac{\phi}{f\sqrt{N}} \right), \quad -N/2 < n \leq N/2. \tag{2.11}$$

Note that the sum of these eigenvalues is independent of ϕ for $N > 1$, and therefore the quadratic divergence in V is a constant, independent of ϕ . The sum of the squares of these eigenvalues is also independent of ϕ for $N > 2$, eliminating even the logarithmic divergence. This explicitly confirms our expectations from power-counting. The (UV-finite) potential for ϕ for $N > 2$ is [6]

$$V(\phi) = -\frac{9}{4\pi^2} g^4 f^4 \sum_{n=1}^{\infty} \frac{\cos(2n\sqrt{N}\phi/f)}{n(n^2 N^2 - 1)(n^2 N^2 - 4)} + \text{constant}. \tag{2.12}$$

Here we see the absence of a divergence without cancellations between particles of different statistics. Rather than eliminating the potential entirely, as would happen in supersymmetry, the cancellations here eliminate the dependence on ϕ . That is, the divergent renormalization is to the cosmological constant, rather than the operator of (2.8). This cancellation is guaranteed by our symmetry discussion; nevertheless it is amusing that the contribution from any individual vector boson mass eigenstate is divergent, while the spectrum and couplings are just right to ensure a finite total result.

The spectrum of this theory exhibits an interesting hierarchy of scales. The highest scale is the UV cut-off of the non-linear sigma model $\Lambda = 4\pi f$. We refer to all the physics below this scale as “low energy”. Physics above this scale can’t be addressed within the non-linear sigma model, although we can of course UV complete this theory in a variety of (conventional) ways [3]: “technicolor”, linear sigma model, supersymmetry, etc. The ϕ mass is insensitive to the details of the physics at and above Λ . Consequently, we need not specify exactly what this physics is for our purposes. Below Λ we have a tower of massive vector bosons, extending down in mass from $\sim gf$ to

$\sim gf/N$. For large N the states near the bottom reproduce the spectrum of an extra-dimensional gauge theory compactified on a circle [3], while for small N (say, 3), no extra-dimensional interpretation is possible. Next, the light scalar ϕ has a mass squared $m_\phi^2 \sim g^4 f^2 / (16\pi^2 N^3)$. The gauge coupling of the unbroken $SU(2)$ is g/\sqrt{N} , so this mass squared is a loop factor smaller than the lightest massive vector boson mass squared, $\sim g^2 f^2 / N^2$. Finally, we have a massless $SU(2)$ gauge boson. We refer to physics near or below the ϕ mass as “very low energy”. In the very low energy theory at this point, we have only the ϕ and the massless gauge bosons.

Note that the ϕ mass is what we would expect from the apparent quadratic divergence in the very low energy theory but with a cutoff of only $\sim gf/N$. This is the scale of new physics: the bottom of the tower of massive vector bosons. This physics is entirely perturbative: no strong interactions are required at this scale to cut off the quadratic divergence. This is to be contrasted with the expectation from pseudo-Nambu–Goldstone bosons in QCD or technicolor. For instance, in the limit where the quark masses vanish in QCD, there is a quadratically divergent contribution to the charged pion mass from photon loops in the low-energy pion effective theory. The pion mass is then of order $e/4\pi$ times $\Lambda \sim 4\pi f_\pi$, the cutoff of the pion effective theory. On the other hand, in our effective theory, there are *no* divergent contributions, the Higgs mass we compute is insensitive to the detailed physics at $4\pi f$, and the result is smaller by an additional factor of $g/(4\pi\sqrt{N})$. In the full theory, this is obvious, as the accidental symmetry prevents the appearance of a counterterm that could absorb the quadratic divergence. But in the very low energy theory, the lightness of ϕ looks miraculous. It is this that makes ϕ an interesting starting point for a model of the Higgs.

In our realistic models the scale of new perturbative physics will be near 1 TeV, while the scale where the non-linear sigma model description breaks down will be parametrically larger, numerically between 10 and 100 TeV. This is similar to what happens in composite Higgs models [2], but the difference here is that we will not require any fine tuning to maintain this ratio of scales.

What was essential for this mechanism to work? The light scalar descended from a “chain” of non-linear sigma model fields, a “non-local” object in the space of gauge theories. Since the gauge interactions themselves *are* local in this space, the only operator that gives rise to the scalar mass must involve the whole chain. This operator is then of very high scaling dimension, and is generated with a finite coefficient. We refer to naturally light scalars of this kind as “chain scalars”.

3. Realistic theories

In order to describe the Higgs field in the Standard Model we need a scalar transforming as a doublet under $SU(2)$, rather than in the adjoint representation. Furthermore, we need a potential that breaks the electroweak symmetry and leaves the physical Higgs scalar somewhat heavier than the Z . This requires a negative mass squared and a substantial quartic self-coupling for the Higgs field. Finally, we need an order one Yukawa coupling to the top quark. We need to incorporate all these features without reintroducing a quadratic divergence for the ϕ mass squared.

Let us first try to get our “chain” scalars out of the adjoint representation of the low-energy gauge group. This can happen if the theory distinguishes different components of the adjoint. This is only possible if the low-energy gauge group is reduced. Consider, for instance, a condensed moose diagram with $SU(3)$ gauge symmetries on the sites $i = 2, \dots, N$, with only the $SU(2) \times U(1)$ subgroup of $SU(3)$ leaving T_8 invariant at $i = 1$. All the link variables continue to be 3×3 special unitary matrices, but the matrix \mathbf{g}_1 resides only in the $SU(2) \times U(1)$ direction, so that \mathbf{g}_1 commutes with T_8 . In the continuum 5D picture, this is a five-dimensional theory with an $SU(3)$ gauge symmetry in the bulk, together with a “brane” where the gauge symmetry is reduced to $SU(2) \times U(1)$.

The fluctuations of the U 's now higgs the theory down to $SU(2) \times U(1)$. The 8 components of the chain scalar decompose under this $SU(2) \times U(1)$ as $\mathbf{3}_0 \oplus \mathbf{1}_0 \oplus \mathbf{2}_{1/2} \oplus \mathbf{2}_{-1/2}$. (This normalization of the $U(1)$ corresponds to the decomposition $\mathbf{3} \rightarrow \mathbf{2}_{1/6} \oplus \mathbf{1}_{-1/3}$.) The last two have the quantum numbers of the standard model Higgs field and its complex conjugate. Since the gauge symmetry no longer relates

these different components, they can pick up different masses. More explicitly, the reduced gauge symmetry on the first site allows additional operators in the theory. Since \mathbf{g}_1 commutes with T_8 , the link variable U_1 and $T_8 U_1$ have identical transformation properties under all symmetries, as do U_N and $U_N T_8$.

As emphasized in [2], to build a Higgs, it is not enough to ensure that it has a small mass. It must also have quartic self-interactions that are large compared to its mass squared over the cut-off squared. The Coleman–Weinberg interactions that produce a small ϕ mass also produce quartic interactions, but these are suppressed by a similar factor. Thus we also need some other source for a quartic potential for the Higgs fields. But the additional self-interactions must not disturb the crucial cancellation of quadratic divergences. Again we can take inspiration from higher-dimensional physics, in which the ϕ is related to a gauge field. Because gauge boson self-interactions contain non-derivative terms, we should be able to build non-derivative interactions for the ϕ . Suppose, for example, that we start in 6D with an $SU(3)$ gauge theory. Then the action contains a piece $\text{tr } F_{56}^2$, that yields a quartic potential

$$\text{tr}([A_5, A_6]^2) \quad (3.1)$$

for the zero modes of A_5, A_6 in the low-energy theory. Because of the higher-dimensional gauge invariance, this should not introduce any divergent masses for the zero modes.

What is the analog of this operator in the condensed moose language? Consider a condensed moose diagram that is the discretization of a torus with $N \times N$ sites, labeled by integers (i, j) . The sites i and $i + N$ are identified, as are j and $j + N$. We also have link fields $U_{i,j}$ between the sites (i, j) and $(i, j + 1)$, and $V_{i,j}$ between (i, j) and $(i + 1, j)$. Finally, we add the “plaquette” operators to the action:

$$-\sum_{i,j} \lambda_{i,j} f^4 \text{tr}(U_{i,j} V_{i,j+1} U_{i+1,j}^\dagger V_{i,j}^\dagger) + \text{h.c.} \quad (3.2)$$

Some of the Nambu–Goldstone bosons are eaten in higgsing the gauge group down to the diagonal $SU(3)$. Others become massive through the plaquette potential. For any values of the $\lambda_{i,j}$ two massless multiplets remain, the analog of the continuum zero modes. It is easy to check that these modes correspond

to

$$U_{i,j} = e^{iu/(fN)}, \tag{3.3}$$

$$V_{i,j} = e^{iv/(fN)}, \tag{3.4}$$

i.e., uniform link variables in the two directions. With this normalization, u, v are canonically normalized fields. For these modes the plaquette action expands to the quartic potential

$$\begin{aligned} &\text{constant} + \frac{\lambda}{N^2} \text{tr} | [u, v] |^2 + \dots, \\ \lambda \equiv &\frac{1}{N^2} \sum_{i,j} \text{Re} \lambda_{i,j}. \end{aligned} \tag{3.5}$$

As expected from the gauge theory analog, the spurious symmetries of the theory are enough to guarantee the absence of divergences for the radiative corrections to these scalar masses for $N > 2$, and only logarithmic divergences for $N = 2$.

Note that the form of the self-interactions in (3.5) is the trace of the square of a commutator. This special form is required to avoid quadratic divergences, and it is an obvious reminder of the analog, (3.1), in the six-dimensional gauge theory. In the very low energy theory additional couplings will be generated through low-energy renormalization.

In order to get the Higgs out of the adjoint representation, we replace the gauge group at one of the sites, say (1, 1), with $SU(2) \times U(1)$. It is easy to show that the classically massless scalar multiplets continue to be those of the form (3.3). As before, the reduced gauge symmetry allows additional operators, in this case involving an insertion of T_8 in the four plaquette terms that touch the (1, 1) site. For instance,

$$(\alpha + i\beta) f^4 \text{tr} T_8 U_{1,1} V_{1,2} U_{2,1}^\dagger V_{1,1}^\dagger + \text{h.c.} \tag{3.6}$$

For the zero modes (3.3) this becomes a mass term

$$-i\beta \frac{f^2}{N^2} \text{tr} T_8 [u, v] + \dots \tag{3.7}$$

If this operator is included with a large coefficient, then there is a large tree-level mass term for the scalars in the theory. However, note that this operator is odd under the interchange $U_{i,j} \leftrightarrow V_{j,i}$. If we impose the symmetry $U_{i,j} \leftrightarrow V_{j,i}$ on the theory, then it is technically natural for β to be small compared to the $\lambda_{i,j}$.

We now have all the ingredients to construct a realistic theory of electroweak symmetry breaking. It

is convenient to group u, v into the matrix

$$\mathcal{H} = \frac{u + iv}{\sqrt{2}} = \begin{pmatrix} \varphi + \eta & h_1 \\ h_2^\dagger & -2\eta \end{pmatrix}, \tag{3.8}$$

where φ, η are complex fields in the $\mathbf{3}_0, \mathbf{1}_0$ representation of $SU(2) \times U(1)$, respectively, and h_1, h_2 have the quantum numbers $\mathbf{2}_{1/2}$ of the standard model Higgs. The quartic potential is

$$\begin{aligned} \lambda \text{tr} [\mathcal{H}, \mathcal{H}^\dagger]^2 &= \lambda \text{tr} (h_1 h_1^\dagger - h_2 h_2^\dagger)^2 \\ &\quad + \lambda (h_1^\dagger h_1 - h_2^\dagger h_2)^2 \\ &\quad + \text{terms involving } \varphi, \eta. \end{aligned} \tag{3.9}$$

Note the similarity of this quartic potential to the one in the supersymmetric standard model. Here, it follows from the special form (3.5).

The Coleman–Weinberg potential generates positive mass squared for all of φ, η, h_1, h_2 . We can also add other operators, such as $\text{tr}(U_{2,1} U_{2,2} \dots U_{2,N}) + \text{h.c.} + (U_{i,j} \rightarrow V_{j,i})$, with small coefficients (since these break the spurious global symmetries it is technically natural for their coefficients to be small). These are of dimension N and therefore do not affect our power counting analysis for the finiteness of the Higgs mass for $N < 5$; and they also preserve the $U \leftrightarrow V$ symmetry. They give rise to the same positive squared mass for all of h_1, h_2, φ, η . In any case, in order to obtain a realistic theory, we need to distinguish between h_1 and h_2 . The reason is familiar from similar considerations in the supersymmetric standard model: since the quartic potential for h_1, h_2 has a flat direction where $|h_1| = |h_2|$, we must have $m_{h_1}^2 + m_{h_2}^2 > 0$ in order not to run away along this flat direction. If the symmetry between h_1, h_2 is unbroken, this forces both m^2 's to be positive and there can be no electroweak breaking. Fortunately, the operator in (3.7) distinguishes between h_1, h_2 . In fact,

$$\begin{aligned} -i\beta \frac{f^2}{N^2} \text{tr} T_8 [u, v] &= \beta \frac{f^2}{N^2} \text{tr} T_8 [\mathcal{H}, \mathcal{H}^\dagger] \\ &= \beta \frac{f^2}{N^2} (h_2^\dagger h_2 - h_1^\dagger h_1). \end{aligned} \tag{3.10}$$

This operator can make one of the masses, say $m_{h_1}^2$, negative, while keeping all the others positive. As we have seen, there is also an $O(1)$ quartic Higgs coupling. To give rise to a small Higgs vev, the coefficient β in (3.7) must be chosen so that this contribution to the h masses is the same order

of magnitude as the Coleman–Weinberg and other contributions. Thus the masses of h_2 and the rest of the \mathcal{H} multiplet are expected to be of the same order of magnitude as the Higgs. But no special fine tuning is required — all these masses are safe from quadratic divergences. There is then a wide range of parameters for which $SU(2) \times U(1)$ is broken in the correct way by the vacuum expectation value of h_1 .⁶

We have succeeded in triggering electroweak symmetry breaking in a natural way. What aspect of the phenomenology of this model can be used to check our mechanism for stabilizing the electroweak hierarchy? In the very low energy theory, below a TeV, we will see not only the Higgs, but the other pseudo-Nambu–Goldstone bosons. The precise spectrum is model dependent, but the existence of a light h_2 and the specific quartic interaction (3.9) is a robust prediction of this mechanism. To make the Higgs much lighter than the h_2 multiplet requires a fine-tuning. At TeV and higher energies, we must see the non-linear sigma model structure with local couplings in theory space. This would reveal itself through low-energy theorems for the scattering of the Nambu–Goldstone bosons, in this case the Higgs fields, h_2 and the longitudinal components of the additional vector bosons at multi-TeV energies.

4. Fermions

For a realistic model, we need fermions with Yukawa couplings to the Higgs. Like gauge couplings and quartic Higgs self-couplings, large Yukawa couplings generically induce a quadratically divergent mass squared for the Higgs boson in the low-energy theory. But this divergence can also be avoided using “locality” in theory space. Before constructing the appropriate local interactions, let us ask what the analogue of the Higgs–Yukawa coupling looks like in our language. As a preliminary example, consider the one-dimensional chain with $SU(3)$ gauge symmetries on the sites $i = 2, \dots, N$ and $SU(2) \times U(1)$ at $i = 1$. We also introduce the standard model fermions with their usual quantum numbers under

⁶ Other terms in (3.6) besides (3.7) have interesting phenomenological consequences, for example, giving rise to a small vev for φ and an (acceptably small) contribution to the T parameter.

$SU(3)_{\text{color}} \times SU(2) \times U(1)$: $Q \sim (\mathbf{3}, \mathbf{2})_{1/6}$, $U^c \sim (\bar{\mathbf{3}}, \mathbf{1})_{-2/3}$, $D^c \sim (\bar{\mathbf{3}}, \mathbf{1})_{+1/3}$, $L \sim (\mathbf{1}, \mathbf{2})_{-1/2}$, $E^c \sim (\mathbf{1}, \mathbf{1})_{+1}$. The Yukawa couplings to the chain “Higgs” field can arise from the gauge invariant operator

$$(Q \ 0) U_1 \cdots U_N \begin{pmatrix} 0 \\ 0 \\ U^c \end{pmatrix} \quad (4.1)$$

for the up Yukawa couplings, and

$$\begin{pmatrix} 0 & 0 & D^c \end{pmatrix} U_1 \cdots U_N \begin{pmatrix} Q \\ 0 \\ 0 \end{pmatrix}, \\ \begin{pmatrix} 0 & 0 & E^c \end{pmatrix} U_1 \cdots U_N \begin{pmatrix} L \\ 0 \\ 0 \end{pmatrix} \quad (4.2)$$

for the down and charged lepton Yukawa couplings.

However, because these operators correspond to “non-local” interactions, adding them to the theory would give rise to a quadratic divergence for the Higgs mass. This may not be a problem for the Yukawa couplings for the light generations, since the coefficient of the quadratic divergence is then easily small enough so that cutting it off at the non-linear sigma model scale does not disturb the light Higgs. However, this is not the case for the top Yukawa coupling. It is technically natural to eliminate these operators, but then we lose the Yukawa coupling to the fermions. What we would like to do instead is obtain these effective operators in the very low-energy theory, starting with purely local interactions in theory space. This can be done in a by now familiar way. For simplicity we consider only the top Yukawa coupling. We add vector-like fermions χ_i, χ_i^c for $i = 2, \dots, N$, that transform as triplets and anti-triplets under $SU(3)_i$, anti-triplets and triplets under $SU(3)_{\text{color}}$, and have $U(1)$ charges $\mp 1/3$. Note that these fermions now transform under gauge interactions other than the $SU(3)_i$ at their site. Together with mass terms, these χ ’s have nearest-neighbor couplings through the U ’s:

$$y_1 f (Q \ 0) U_1 \chi_2 + \sum_{i=2}^{N-1} \chi_i^c (y_i f \chi_i - y_i^c f U_i \chi_{i+1}) \\ + y_N f \chi_N^c U_N \begin{pmatrix} 0 \\ 0 \\ U^c \end{pmatrix}. \quad (4.3)$$

Integrating out the massive χ, χ^c leaves the massless fields Q, U^c coupled to the Higgs in the low-energy

theory. However, with these local interactions, our power-counting analysis guarantees the absence of all divergences in the Higgs mass for $N > 2$, and only logarithmic divergences for $N = 2$.

Finally, we want to introduce fermions in the $N \times N$ site model of the previous section, where the Higgs triggers electroweak symmetry breaking. This can be done in a number of ways. For instance, we can have a chain of fermions as in the previous paragraph, in both the U and V directions symmetrically, to preserve the $U \leftrightarrow V$ symmetry.

Note that the operators (4.1), (4.2) may be used to generate the Yukawa couplings for the 2 light generations. In this case the only interactions which break the $U(2)^5$ flavor symmetry are the Yukawa couplings themselves. This guarantees the absence of dangerous flavor changing neutral currents via the GIM mechanism.

5. Conclusions

We have seen that if the Higgs field descends from a chain of links in theory space, we can trigger electroweak symmetry breaking in a way that remains perturbative and insensitive to high-energy details up to a cut-off scale much larger than a TeV without the need for any fine-tuning. The radiative corrections to the Higgs mass are controllably small, without relying on supersymmetry or strong dynamics at the TeV scale. Notice, in particular, we have included, in a natural way, a set of operators with varying sizes. A fundamental theory above the scale where our non-linear sigma model description breaks down will determine which operators appear in the low energy theory. However, we have seen that it is natural to allow some “local” operators with large coefficients, while higher-dimensional “non-local” operators have small ones.

But where does “locality” come from? Normally, locality of interactions in position space is simply taken for granted. However, as we emphasized in [3], there is an intimate connection between physical space and theory space, where locality can have a deeper origin. For instance, in the simple constructions of [3], higher-dimensional locality is a consequence of the renormalizability of the 4D gauge theory that dynamically generated the extra dimension. We expect that

similar considerations will provide a deeper explanation of the pattern of local operators in our present constructions, once our non-linear sigma model is UV completed in a renormalizable theory at high energies, above ~ 10 to 100 TeV.

Our models provide a realistic theory of electroweak symmetry breaking, although they are by no means the unique implementation of our central mechanism. For instance, elegant “orbifold” theories can be constructed where the only light scalars are $SU(2)$ doublets. But it is crucial that there be a light h_2 partner of the Higgs doublet with the squared commutator self-interaction of (3.9).

The theories we have constructed have a rich and novel phenomenology at TeV energies. In addition to a light Higgs, they have a distinctive spectrum of new scalar, fermionic and vector particles with perturbative couplings (for moderate N). In this decade, experiment will help us determine whether or not extended objects in theory space are relevant to unraveling the mystery of electroweak symmetry breaking.

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